PACE: Fully Parallelizable BFT from Reproposable Byzantine Agreement

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ABSTRACT

The classic asynchronous Byzantine fault tolerance (BFT) framework of Ben-Or, Kemler, and Rabin (BKR) and its descendants rely on reliable broadcast (RBC) and asynchronous binary agreement (ABA). However, BKR does not allow all ABA instances to run in parallel, a well-known performance bottleneck. We propose PACE, a generic framework that removes the bottleneck, allowing fully parallelizable ABA instances. PACE is built on RBC and *reproposable ABA* (RABA). Different from the conventional ABA, RABA allows a replica to change its mind and vote twice. We show how to efficiently build RABA protocols from existing ABA protocols and a new ABA protocol that we introduce.

We implement six new BFT protocols: three in the BKR framework, and three in the PACE framework. Via a deployment using 91 replicas on Amazon EC2 across five continents, we show that all PACE instantiations, in both failure-free and failure scenarios, significantly outperform their BKR counterparts, and prior BFT protocols such as BEAT and Dumbo, in terms of latency, throughput, latency vs. throughput, and scalability.

KEYWORDS

asynchronous BFT, binary agreement, blockchain, fault tolerance, RABA

1 INTRODUCTION

Byzantine fault tolerance (BFT) is widely viewed as the model for permissioned blockchains. Among BFT protocols, completely asynchronous BFT protocols [4, 12, 16, 34, 39, 43] have received renewed attention because of its intrinsic robustness. Indeed, having long been viewed as a "theoretical" approach, several recent asynchronous BFT systems—such as HoneyBadgerBFT [38], BEAT [29], Dumbo [31], and EPIC [35]—have shown their performance becomes comparable to their partially synchronous counterparts.

Efficient asynchronous BFT protocols may be roughly divided into two categories: the BKR (Ben-Or, Kelmer, and Rabin) paradigm [12] including HoneyBadgerBFT, BEAT, and EPIC, and the CKPS (Cachin, Kusawe, Petzold, and Shoup) paradigm [16] including SINTRA [18] and Dumbo. Both paradigms have their benefits and drawbacks: the BKR framework is information-theoretically (IT) secure (if assuming an IT common coin protocol) and achieves quantum safety (as defined in [32]); it has an $O(\log n)$ running time. The CKPS framework is only computationally secure and relies on less well-established cryptographic pairing assumptions; it has an $O(1)$ running time but a large hidden constant.

Neglecting the security model, even just considering performance, the "common belief" that there is no "one-size-fits-all" BFT protocol has, thus far, remained true for the case of asynchronous BFT. For instance, a recent (state-of-the-art) asynchronous BFT, Dumbo, largely follows but refines CKPS by reducing its communication complexity, at the price of four more all-to-all communications and $O(n^3)$ pairing-based threshold signatures. Dumbo, however, has 14 steps even in the best-case scenario (where the ABA instance terminates in one round). The protocols following the BKR paradigm terminate in $O(\log n)$ rounds but only 6 steps in the best-case scenario. Our experiment shows that BEAT, for instance, outperforms Dumbo when $n \leq 46$, but is less efficient than Dumbo for larger n's.

This work introduces a new BFT framework called PACE that removes a well-known performance bottleneck in the BKR paradigm, and therefore improves upon the BKR framework and applications along this line of research (e.g., all BKR descendants, asynchronous distributed key generation [27, 33], interactive consistency [11]). We show that our PACE instantiations significantly outperform their BKR counterparts, and existing BFT protocols such as BEAT and Dumbo, in both failure and failure-free scenarios.

The BKR bottleneck. The BKR paradigm reduces asynchronous BFT to RBC and ABA. In each epoch, all replicas first run an RBC phase to reliably broadcast their proposals. Then they run an ABA phase, where n parallel ABA instances are invoked. The i-th ABA instance agrees on whether the proposal of replica p_i has been delivered in the RBC phase: Upon RBC delivery of a proposal from p_j, the replica proposes 1 to the j-th ABA instance. If a correct replica p_j decides 1 for the i-th ABA instance, the proposal from p_i is delivered. Otherwise, the proposal is not included. The BKR paradigm requires that if a replica has not received some proposals during the RBC phase, the replica *abstains* from proposing 0 until $n - f$ ABA instances terminate with 1. This method ensures that proposals from at least $n - f$ replicas are delivered. The BKR paradigm, however, breaks the parallelism of the "RBC+ABA" structure. As shown in Figure 1a, the ABA phase has two subphases: replicas have to wait until at least $n - f$ ABA instances terminate with 1 and then invoke the remaining ABA instances with 0.

This is a well-known bottleneck for the BKR paradigm repeatedly pointed out by prior works [10, 11] and recently emphasized by HoneyBadgerBFT [38, Section 4.4]. In addition, both Dumbo and BEAT experimentally validate this bottleneck via performance breakdown for the building blocks. The two-subphase pattern not only adds significant latency but causes transaction congestion.

A well-known "naive attempt" that does not work (and was recently recalled in [38]) would be that each replica waits for the first $n-f$ RBCs completed, and then propose 1 for the ABA instances corresponding to those completed and propose 0 for the others. However, RBC instances completed for correct replicas may be different. As ABA ensures the decided value is 1 only if all correct

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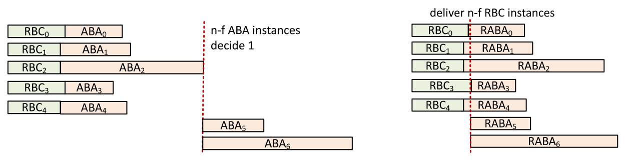
protocols	framework	ABA/ RABA	ABA/ RABA used	coin type	IT secure (using IT common coin)?	quantum safety?	cryptographic assumption
Dumbo [31]	CKPS	ABA	Cobalt [37]	regular			pairing
HoneyBadgerBFT [38]	BKR	ABA	MMR [41]	regular	√	√	pairing
BEAT-MMR [29]	BKR	ABA	MMR [41]	regular	√	√	CDH
BEAT-CrainH (this paper)	BKR	ABA	CrainH [25]	high threshold	√	√	DDH
BEAT-CrainL (this paper)	BKR	ABA	CrainL [25, 28]	regular	√	√	CDH
BEAT-Pillar (this paper)	BKR	ABA	Pillar★	regular	√	√	CDH
PACE-CrainH-R (this paper)	PACE	RABA	CrainH-R★	high threshold	√	√	DDH
PACE-CrainL-R (this paper)	PACE	RABA	CrainL-R★	regular	√	√	CDH
PACE-Pisa (this paper)	PACE	RABA	Pisa★	regular	√	√	CDH

Table 1: Comparison among asynchronous BFT protocols. "Coin type" includes regular, low threshold ($f + 1$) common coins and high threshold ($n - f$) common coins. "IT security" means information-theoretic security. HoneyBadgerBFT and BEAT, strictly speaking, do not attain IT security even if assuming IT secure common coins; they can be made IT secure if following the technique of EPIC in selecting transactions, so we mark "yes" for them for IT security. Quantum safety is defined in DAG-Rider [32]; quantum security = quantum safety + quantum liveness. ★New ABA/RABA protocols in this work.



(a) BKR (e.g., HoneyBadgerBFT, BEAT). **(b) PACE.**

Figure 1: BKR vs. PACE. Assume there are 7 replicas with 2 failures. BKR has two ABA subphases: replicas have to wait for the slowest ABA in the first subphase to terminate before invoking the second subphase. PACE uses reproposable ABA (RABA) to remove the two-subphase bottleneck.

replicas unanimously propose 1, the transactions delivered may be empty.

PACE explained. In our work, we propose a new framework, PACE, that makes the above "naive attempt" work and removes the two-subphase bottleneck. We achieve this by replacing ABA with *reproposable* ABA (RABA), a new binary agreement primitive.

Our key observation is that in the naive attempt, if $n - f$ correct replicas deliver some proposals in $n - f$ RBC instances, there must exist at least $f + 1$ correct replicas that will propose 1 for at least $f + 1$ ABA instances. For these specific ABA instances, we want to guarantee that all these ABA instances will indeed decide 1. The property is called *biased validity*. Meanwhile, we need to make ABA protocols syntactically *reproposable*, in the sense that a replica who previously voted 0 (maybe immaturely) may change its mind to vote 1 (if the replica later delivers the corresponding RBC instance). The property (called *biased termination*) ensures that all RABA instances can terminate.

We offer generic strategies that can convert various efficient ABA protocols to efficient RABA protocols. Interestingly, the "biased" features of RABA provide a "fast" path for RABA to decide, further reducing latency and improving throughput.

1.1 Our Contributions

RABA. We suggest *reproposable ABA* (RABA), a new ABA primitive allowing replicas to change votes if needed. We formally describe the properties of RABA. We view RABA as a first-class distributed computing primitive.

PACE: A new framework for BFT, parallel ABA, interactive consistency, etc. We design a new framework for asynchronous BFT that use RBC and RABA in a black box manner. The framework leads to the first fully parallelizable asynchronous BFT protocol, allowing *all* RABA instances to run in a strictly concurrent way. The improvement can both increase the throughput and reduce the latency, effectively removing the BKR two-subphase bottleneck. PACE can also improve interactive consistency [11], the asynchronous common subset (ACS) framework and applications using ACS (e.g., asynchronous distributed key generation [27, 33]).

ABA and RABA protocols. We provide generic strategies that transform efficient ABA protocols to RABA protocols. All our RABA protocols have a fast path for RABA protocols to terminate. In particular, we demonstrate the transformation for the classic CKS ABA [17], Cobalt ABA [37], and the two protocols from Crain's work—CrainH (using high threshold common coins) and CrainL (using low threshold common coins) [25], and an ABA protocol that we introduce in the paper (called Pillar). For CrainL, we use the restated version with the good-case-coin-free property [28]. For Pillar, it is a new ABA protocol that has the same steps as CrainH but uses more efficient low threshold common coins. The resulting RABA protocols are called CKS-R, Cobalt-R, CrainH-R, CrainL-R, and Pisa, respectively. These ABA and RABA protocols provide interesting efficiency trade-offs.

Practical contributions. As shown in Table 1, we implement three ABA protocols (CraintH, CrainL, Pillar) and three RABA protocols (CrainH-R, CrainL-R, Pillar). Correspondingly, we implement six new BFT protocols: three in the BKR framework (BEAT-CrainH, BEAT-CrainL, BEAT-Pillar), and three in the PACE framework (PACE-CrainH-R, PACE-CrainL-R, PACE-Pisa). We extensively evaluate the performance for these six new protocols and compare them with existing protocols, including BEAT-MMR, BEAT-Cobalt, and Dumbo. Via an EC2 deployment using 91 replicas from five continents, we demonstrate that all PACE instantiations, in both failure-free and failure scenarios, dramatically outperform their BKR counterparts, and prior BFT protocols such as BEAT and Dumbo, in terms of all metrics (latency, throughput, latency vs. throughput, and scalability). Let us highlight some evaluation results:

- All PACE instantiations significantly outperform any of the BRK instantiations. Even the slowest PACE instantiation is far more efficient than all the other protocols.
- PACE-Pisa and PACE-CrainL-R are consistently more efficient than PACE-CrainH-R. PACE-Pisa slightly outperforms PACE-CrainL-R, except that when the system is of medium size, the two protocols offer interesting trade-offs. So below we use PACE-Pisa for some concrete comparison for simplicity.
- For $f = 1$, PACE-Pisa achieves 1.77x the throughput of BEAT-Cobalt and 5.1x the throughput of Dumbo. For $f = 30$, PACE-Pisa achieves 3.6x the throughput of BEAT-Cobalt and 1.66x the throughput of Dumbo.
- PACE-Pisa offers an impressive latency metric (when there is no contention) in both LAN and WAN environments. In WANs, the latency of PACE-Pisa is between 1/5 and 1/2 of that of the BKR protocols and between 1/3 and 1/2 of that of Dumbo. In LANs, Dumbo has 9x the latency of PACE-Pisa for $f = 1$.
- All PACE protocols are highly robust against various crash and Byzantine failure scenarios. Each PACE protocol outpaces its BKR counterpart in all failure scenarios. Remarkably, among all failure scenarios, the slowest PACE instantiation remains more efficient than the fastest BEAT protocol in its failure-free scenario.

All protocols introduced in the paper rely on well-studied, standard, and pairing-free cryptographic assumptions (CDH or DDH for elliptic curves), achieving standard 128-bit security.

2 SYSTEM MODEL AND DEFINITIONS

We consider distributed computing protocols, where f out of n replicas may fail arbitrarily (Byzantine failures). The protocols we consider in this work (ABA, BFT, and RBC) assume $f \leq \lfloor \frac{n-1}{3} \rfloor$, which is optimal. We consider completely asynchronous systems making no timing assumptions on message processing or transmission delays. A (Byzantine) *quorum* is a set of $\lceil \frac{n+f+1}{2} \rceil$ replicas. For simplicity, we may assume $n = 3f + 1$ and a quorum size of $2f + 1$. In our protocols, we may associate each protocol instance with a unique session identifier sid, tagging each message in the protocol with sid; we may omit these identifiers when no ambiguity arises.

This paper studies BFT protocols. The protocols that we introduce in the paper inherit all features that BKR and its descendants have: quantum safety and information-theoretic (IT) security (if assuming IT common coin). All these protocols rely on the standard and well-studied Computational Diffie-Hellman (CDH) or Decisional Diffie-Hellman (DDH) assumptions. In contrast, protocols derived from the CKPS paradigm [16] (e.g., Dumbo) achieve computational security only and use (stronger) pairing assumptions. Our implementations tolerate static corruption, where the adversary needs to choose the set of corrupted replicas before the execution of the protocol. But our protocols can easily achieve adaptive security, if using adaptively secure common coin protocols [9, 35].

We use BFT and (Byzantine) atomic broadcast interchangeably. Syntactically, in BFT, a replica *a-delivers* (atomically deliver) *transactions*, each *submitted* by some client. The client computes a final response to its submitted transaction from its responses from replicas. The correctness of a BFT protocol is specified as follows:

- **Agreement**: If any correct replica *a-delivers* a transaction tx, then every correct replica *a-delivers* tx.
- **Total order**: If a correct replica *a-delivers* a message tx before *a-delivering* tx', then no correct replica *a-delivers* a message tx' without first *a-delivering* tx.
- **Liveness**: If a transaction tx is *submitted* to all correct replicas, then all correct replicas eventually *a-deliver* tx.

Asynchronous (binary) Byzantine agreement (ABA). An ABA abstraction is specified by *propose* and *decide*. Each replica proposes an initial binary value (*vote*) for consensus and replicas will decide on some value. ABA should satisfy the following properties:

- **Validity**: If all correct replicas *propose* v, then any correct replica that terminates *decides* v.
- **Agreement**: If a correct replica *decides* v, then any correct replica that terminates *decides* v.
- **Termination**: Every correct replica eventually *decides* some value.
- **Integrity**: No correct replica *decides* twice.

RBC. We review the definition of Byzantine reliable broadcast (RBC). A RBC protocol is specified by *r-broadcast* and *r-deliver* such that the following properties hold:

- **Validity**: If a correct replica p *r-broadcasts* a message m, then p eventually *r-delivers* m.
- **Agreement**: If some correct replica *r-delivers* a message m, then every correct replica eventually *r-delivers* m.
- **Integrity**: For any message m, every correct replica *r-delivers* m at most once. Moreover, if a replica *r-delivers* a message m with sender s, then m was previously *r-broadcast* by replica s.

The paper uses AVID RBC [19] that achieves $O(n|m| + \lambda n^2 \log n)$ communication, where λ is a security parameter. In both BKR and PACE frameworks, the communication is dominated by n RBC protocols and is $O(Ln^2 + \lambda n^3 \log n)$ (L is the input size).

Throughout the paper, we also use *best-effort broadcast*, or simply *broadcast*, where a sender multicasts a message to all replicas.

Common coins and thresholds. The common coin protocol outputs a binary value at each correct replica [42]. These protocols can be divided into protocols with the regular (low) thresholds (i.e., $f + 1$) and protocols with the high thresholds (i.e., $2f + 1$). Protocols from both regular coins [37, 40] and high threshold coins [17, 24, 25] have been proposed.

It is preferred to use regular common coin protocols over high threshold common coin protocols from both theoretical perspective and practical perspective. First, without assuming a trusted setup, it is more expensive to build decentralized key generation protocols for high threshold common coins than for regular common coins, as high threshold asynchronous verifiable (complete) secret sharing is more expensive than the regular one [6, 27]. Second, even assuming the trusted setup model, regular common coin protocols are easier to construct. For instance, the most efficient common coin protocols due to Cachin, Kursawe, and Shoup [17] include a regular common coin protocol using the Computational Diffie-Hellman (CDH) assumption and a high threshold common coin protocol that must use the (stronger) Decisional Diffie-Hellman (DDH) assumption. Third, in the trusted setup model, regular common coin protocols are less efficient than high threshold protocols, because for high threshold, one would need to wait for more shares and need to combine more shares to generate the coins.

Steps; rounds. We use the standard definition of (communication) steps [15], where a step consists of receiving a message from another process, executing a local computation, and sending a message to some process. For instance, server-side PBFT has three steps. ABA protocols proceed in rounds, where each round has a fixed number of steps. So the total number of steps for ABA is a product of #rounds and #steps per round.

3 CANDIDATE ABA PROTOCOLS FOR PACE: EXISTING PROTOCOLS AND PILLAR

In this section, we first review existing, practical ABA protocols relying on authenticated channels, focusing on MMR ABA [40] and its descendants. We then present in Figure 2 Pillar, a new ABA protocol assuming authenticated channels. We also briefly review CKS ABA [17] and Crain-Sig20 ABA [24] that use threshold signatures. All these ABA protocols described in the section are candidate protocols for our PACE framework and can in fact be efficiently converted to RABA protocols.

3.1 Existing ABA Protocols

ABA protocols proceeds in rounds, where each round has a fixed number of steps.

The ABA protocol by Mostefaoui, Moumen, and Raynal (MMR) [40] (MMR ABA) is the first constant-round ABA that relies on authenticated channels only. In each round, MMR ABA has 2 or 3 steps (without counting the step for common coin). Asynchronous BFT protocols, such as HoneyBadgerBFT [38] and BEAT [29], in their proceeding versions, utilize MMR ABA.

When describing MMR ABA and other protocols, we follow prior notations like, e.g., $\mathsf{bval}_r(a)$, where $\mathsf{bval}_r()$ is a message pattern with a round r, and a is the message transmitted. In MMR ABA, each round r has two phases: a dispersal phase and an agreement phase. In the dispersal phase, every replica broadcasts its value est_r via a $\mathsf{bval}_r(est_r)$ message. Upon receiving $f+1$ $\mathsf{bval}_r(v)$, a replica also broadcasts a $\mathsf{bval}_r(v)$ if it has not done so. Upon receiving $n-f$ $\mathsf{bval}_r(v)$, a replica adds v to a local set bin_values_r. Each replica broadcasts an $\mathsf{aux}_r(v)$ message for the first value added to bin_values_r and then enters the agreement phase. Upon receiving $n-f$ $\mathsf{aux}_r()$ messages, replicas generate a common coin s_r by querying the common coin protocol coin_r. If a replica receives $n-f$ $\mathsf{aux}_r(b)$ and $b = s_r$, the replica decides b and sets est_{r+1} to b. Otherwise, the replica sets est_{r+1} to s_r. Each replica then enters round $r+1$. After each replica decides, there are two *standard* approaches to terminating the protocol. One approach is that each replica continues to participate in the protocol until reaching round r', where $s_{r'}$ equals the value the replica decides. The other approach is that each replica broadcasts a message to all replicas and each replica terminates the protocol upon receiving $n-f$ such messages. Throughout the paper, we neglect the details of terminating the protocol. The pseudocode of MMR is presented in Appendix A.

MMR ABA, however, has a liveness issue [1, 41, 45]. In particular, the protocol assumes perfect random coins completely independent of the state of all correct replicas at the point when they query the coin. The property cannot be guaranteed by any cryptographic common coin protocols. A malicious network scheduler can force correct replicas to always enter the next round with inconsistent values (i.e., some replicas set est_{r+1} to the common coin value s_r after receiving both $\mathsf{aux}_r(0)$ and $\mathsf{aux}_r(1)$ and some replicas set est_{r+1} to $\bar{s}_r$ after receiving $n-f$ $\mathsf{aux}_r(\bar{s}_r)$).

The journal version of MMR [41, 2nd algorithm] fixed the issue but has 9 to 13 steps for each round. The idea is to repeat the dispersal-agreement pattern four times such that the value each replica decides does not have to be compared with the common coin. The ABA can work for both weak coins and perfect coins. Cobalt ABA [37] provides an alternative solution, having 3 or 4 steps in each round. Recent BFT implementations including EPIC [35] and Dumbo [31] use Cobalt ABA.

Crain [25, 1st algorithm] (denoted as CrainL) optimized the MMR journal protocol by having 5 to 7 steps in each round. The core idea is that the dispersal-agreement pattern only has to be repeated twice instead of four times. As in MMR journal, CrainL work for weak coins and perfect coins. A recent work pointed out that CrainL can have a "good-case-coin-free" property which leads to a fast path to decide values [28]. We use the restated version of CrainL from [28].

In the same work, Crain [25, 2nd algorithm] (denoted as CrainH) also fixed the issue of MMR ABA assuming authenticated channels and has 2 or 3 steps in each round. However, CrainH uses a high threshold perfect common coin protocol which, as we have argued in Sec. 2, is less efficient, and may rely on a stronger assumption.

For all these ABA protocols, if using perfect coins, the expected number of rounds for replicas to reach a state such that a decision can be made is 2. Then replicas continue to execute the protocol until they decide. For MMR and its descendants (Cobalt, CrainH, Pillar), as replicas have to compare its value with the common coin before they decide, another 2 rounds are expected and the total expected number of rounds to terminate is 4. For CKS, the MMR journal protocol, and CrainL, as replicas do not have to compare their values with the coin, replicas are expected to decide in another one round and the expected number of rounds for these protocols to terminate is 3.

Table 2 summarizes some representative ABA protocols terminating in an expected constant number of rounds. We divide these protocols into two categories: ABA assuming authenticated channels only and ABA requiring the transferability of (threshold) signatures (including CKS ABA and Crain-Sig20 [24]). Jumping ahead, in Sec. 4 and Appendix G, we show how to convert Pillar, CKS, CrainH, CrainL, and Cobalt to RABA.

3.2 Pillar

	signature?	steps/round	rounds	coin type
CKS [17]	yes	2 or 3*	3	high threshold
MMR [40] (insecure)	no	2 or 3	4	regular
MMR [41, 2nd alg]	no	9 to 13	3	regular (weak) coin
Cobalt [37]	no	3 or 4	4	regular
Crain-Sig20 [24, 28]	yes	1 or 2‡	3	high threshold
CrainL [25, 1st alg]	no	5 to 7	3	regular (weak) coin
CrainH [25, 2nd alg]	no	2 or 3†	4	high threshold
Pillar (this work)	no	2 or 3	4	regular

Table 2: Comparison of ABA protocols using common coins. Steps/round denotes number of steps per round (common coin steps not counted). Rounds denote the expected number of rounds. The total number of steps is a product of steps/round and rounds. *CKS has 3 steps in round 0. ‡Crain-Sig20 is a variant of CKS with 2 steps in round 0 and 1 step in round greater than 0.

```
01 upon event $propose(sid, v_i)$
02   if $r = 0$, $est_0 \leftarrow v_i, maj_0 \leftarrow \bot$, start round 0
03 round $r$
04   $bin\_values_r \leftarrow \emptyset, majs \leftarrow \emptyset$
05   broadcast $\mathsf{bval}_r(est_r, maj_r)$                    {1st phase}
06   upon receiving $\mathsf{bval}_r(v, c)$ from $j$
07     $majs \leftarrow majs \cup \{c\}$
08   upon receiving $f + 1$ $\mathsf{bval}_r(b, *)$
09     if $\mathsf{bval}_r(b, maj_r)$ has not been sent
10       broadcast $\mathsf{bval}_r(b, maj_r)$
11   upon receiving $n - f$ $\mathsf{bval}_r(b, *)$
12     $bin\_values_r \leftarrow bin\_values_r \cup \{b\}$
13     if ($r > 0$ and (($b = \bar{s}_{r-1}$ and $\mathsf{V}_1(majs) = b$) or ($b = s_{r-1}$ and $\bar{b} \notin majs$)))
or $r = 0$
14       $\delta_r(b) \leftarrow 1$
15     if $\mathsf{aux}_r(b, *)$ or $\mathsf{aux}_r(\bar{b}, *)$ has not been sent
16       if $\delta_r(b) = 1$, broadcast $\mathsf{aux}_r(b, b)$              {2nd phase}
17       else broadcast $\mathsf{aux}_r(\bot, b)$
18   upon receiving $n - f$ $\mathsf{aux}_r(*, *)$ where the sets of values carried by these
messages are $vals_r$ and $avals_r$; $vals_r$ and $avals_r$ are both subsets of $bin\_values_r$.
19     $s_r \leftarrow \mathsf{coin}_r$
20     if $\mathsf{V}_2(vals_r, b) \geq \lceil \frac{n+f+1}{2} \rceil$         {a quorum of replicas voted for $b$}
21       $est_{r+1} \leftarrow b, maj_{r+1} \leftarrow b$
22       if $b = s_r$, $decide(sid, b)$
23     if $r > 0$ and ($\mathsf{V}_1(vals_r) = \bot$ or $\mathsf{V}_2(vals_r, b) < \lceil \frac{n+f+1}{2} \rceil$)
24       if $\mathsf{V}_2(avals_r, b) \geq \lceil \frac{n+f+1}{2} \rceil$
25         $est_{r+1} \leftarrow b, maj_{r+1} \leftarrow b$
26         if $b = s_{r-1}$ and $b = s_r$, $decide(sid, b)$
27       else if $\{0, 1\} \in avals_r$ and $b = s_{r-1}$
28         $est_{r+1} \leftarrow b$
29         $maj_{r+1} \leftarrow b$
30     if $est_{r+1}$ has not been set yet                  {all other conditions}
31       $est_{r+1} \leftarrow s_r$
32       if $r = 0$, $maj_{r+1} \leftarrow s_r$
33       else $maj_{r+1} \leftarrow \mathsf{majority}(vals_r)$
34     $r \leftarrow r + 1$, continue to the next round
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Figure 2: The Pillar protocol. The code for replica p_i. Broadcast in the code means best-effort broadcast.

Pillar is designed to address the liveness issue of MMR without introducing more steps or using less efficient high threshold common coins. As in MMR ABA and its descendants, each round in Pillar consists of a dispersal phase and an agreement phase. The core idea of Pillar is that instead of having each replica include one value in each message, in Pillar, each message consists of two values. The first value is the "main value" for which each replica votes. The second value carries auxiliary value. The auxiliary value guarantees that correct replicas will only vote for the same value in the agreement phase of each round. This prevents the value voted by each correct replica from being manipulated by an adversary. We introduce some notation to better present the protocol.

- For $b \in \{0, 1\}$, $\bar{b} = \neg b = 1 - b$.
- $*$ in a message may represent $b, \bar{b}$, or $\bot$, where $b \in \{0, 1\}$ and $\bot \notin \{0, 1\}$. For instance, $\mathsf{aux}_r(*, b)$ may represent $\mathsf{aux}_r(b, b)$, $\mathsf{aux}_r(\bar{b}, b)$, or $\mathsf{aux}_r(\bot, b)$. $\mathsf{bval}_r(b, *)$ may represent $\mathsf{bval}_r(b, b)$, $\mathsf{bval}_r(b, \bar{b})$, or $\mathsf{bval}_r(b, \bot)$. We may simply omit $*$ when there is no ambiguity; for example, we may write $\mathsf{aux}_r()$ to denote $\mathsf{aux}_r(*, *)$.
- Let $vals$ be a vector (multiset) consisting of 0, 1, and $\bot$. $\mathsf{V}_1(vals) = b$ if the only value in $vals$ is b for $b \in \{0, 1, \bot\}$.
- $\mathsf{V}_2(vals, b) = t$ if $vals$ includes b only or both b and $\bot$, where $b \in \{0, 1\}$, and the number of b's in $vals$ is t.
- $\mathsf{majority}(vals) = b$ for $b \in \{0, 1\}$, if b is a simple majority in $vals$, i.e., the number of b's is no less than $\lceil (|vals| + 1)/2 \rceil$. $\mathsf{majority}(vals) = \bot$ otherwise.

As illustrated in Figure 2, the Pillar protocol has two message types: $\mathsf{bval}_r()$ and $\mathsf{aux}_r()$, corresponding to the dispersal phase and the agreement phase, respectively.

In each round r, a replica p_i has an input est_r and an auxiliary input maj_r. The est_r value is a binary value (i.e., $est_r \in \{0, 1\}$) and $maj_r \in \{0, 1, \bot\}$. In the first phase (ln 05-17), p_i first broadcasts a $\mathsf{bval}_r(est_r, maj_r)$ message. A correct replica does not change its maj_r within a round. If p_i receives more than $f + 1$ $\mathsf{bval}_r(b, *)$ messages such that b is different from its input, it also broadcasts $\mathsf{bval}_r(b, maj_r)$ (ln 08-10). If a replica receives $2f + 1$ $\mathsf{bval}_r(b, *)$ messages with the same b value (b is either 0 or 1), b is added to a set bin_values_r (ln 11-12).

In addition, p_i checks the maj_r values (which form a set $majs$) in the $\mathsf{bval}_r(b, maj_r)$ messages. Specifically, $\delta_r(b)$ is set as 1 for the following conditions (ln 13-14). If $r = 0$, $\delta_r(b)$ is set as 1 for any $b \in \{0, 1\}$. If $r > 0$, the replica compares b with the common coin value s_{r-1} in round $r - 1$. If $b = s_{r-1}$ and p_i receives only $\mathsf{bval}_r(b, b)$ and $\mathsf{bval}_r(b, \bot)$, $\delta_r(b)$ is set as 1. If $b = \bar{s}_{r-1}$ and p_i only receives $\mathsf{bval}_r(b, b)$ (i.e., $\mathsf{V}_1(majs) = b$), $\delta_r(b)$ is set as 1. After p_i receives $n - f$ $\mathsf{bval}_r()$ messages, it sends $\mathsf{aux}_r(b, b)$ when $\delta_r(b) = 1$ and $\mathsf{aux}_r(\bot, b)$ otherwise (ln 15-17). A correct replica only sends one $\mathsf{aux}_r()$ message in each round.

In the second phase (ln 18-34), replica p_i only accepts an $\mathsf{aux}_r(v_1, v_2)$ message if both v_1 and v_2 are added to bin_values_r ($\bot$ is considered as $\{0, 1\}$). Furthermore, a replica does not accept an $\mathsf{aux}_r(v, \bar{v})$ or $\mathsf{aux}_r(*, \bot)$ message, since these values are not allowed according to our specification. Furthermore, if a replica sets $\delta_r(b) = 1$ and receives $\mathsf{aux}_r(\bar{b}, *)$, it discards the message. Upon receiving $n - f$ $\mathsf{aux}_r()$ messages, p_i queries the common coin protocol and obtain s_r (ln 19). When processing the $\mathsf{aux}_r()$ messages, there are three cases for p_i:

- Case 1 (ln 20-22). If p_i receives a quorum of $\mathsf{aux}_r(b, b)$ messages, p_i compares b with the common coin s_r. If $b = s_r$, p_i decides b. Otherwise p_i set both est_{r+1} and maj_{r+1} as b.
- Case 2 (ln 23-26). If p_i only receives $\mathsf{aux}_r(b, b)$ and $\mathsf{aux}_r(\bot, b)$ and at least a quorum of the messages are of the form $\mathsf{aux}_r(*, b)$, p_i also compares b with both s_{r-1} and s_r. If $b = s_{r-1}$ and $b = s_r$, p_i decides b. Otherwise, p_i uses b for est_{r+1} and maj_{r+1}.
- Case 3 (ln 27-29). If p_i receives $n - f$ $\mathsf{aux}_r(\bot, b)$ and $\mathsf{aux}_r(b, b)$ and $b = s_{r-1}$, p_i uses b for est_{r+1} and maj_{r+1}.

If none of the cases apply and est_{r+1} has not been set yet (ln 30-33), p_i uses the common coin value est_{r+1} for the next round. Furthermore, p_i sets maj_{r+1} to s_r for $r = 0$ and majority($vals_r$) for $r > 0$. Then p_i enters the next round.

Security analysis. There are two key elements in our design. First, each message carries a main value and an auxiliary value. In the $\mathsf{bval}_r()$ step, the auxiliary value maj_r *will never change in the same round*, though a replica is allowed to broadcast $est_r = 1$ and $est_r = 0$. Furthermore, we require that a replica sends an $\mathsf{aux}_r(b, b)$ message if $\delta_r(b) = 1$. In round 0, $\delta_r(b) = 1$ for both $b = 0$ and $b = 1$. But in round $r > 0$, correct replicas will only have $\delta_r(b) = 1$ for at most one value (Lemma D.1). This naturally prevents a network scheduler from making some correct replicas set est_{r+1} as a value different from the common coin coin_r. Second, we ensure that if a correct replica decides in round r, all replicas will have the same est_{r+1} and eventually decide est_{r+1}. This is achieved by having replicas check both values in $\mathsf{aux}_r()$ messages.

Note that in one case (ln 26, Figure 2), we require that $b = s_{r-1} = s_r$, i.e., two consecutive coins with the same value. However, the expected number of rounds for Pillar remains the same as prior works such as MMR, Cobalt, and CrainH. Note in round r, s_{r-1} is already a *fixed* value. In particular, if all correct replicas propose the same value b, replicas are already in a state where they will reach a decision. If correct replicas propose different values, correct processes that do not converge to the same value at the end of each round will use the common coin. The expected number of rounds for replicas to reach a state where they will reach a decision is 2. After that, as replicas have to compare their value with the common before deciding the value, another 2 rounds are expected before replicas decide. In total, the expected number of rounds is 4.

4 REPROPOSABLE ABA (RABA)

4.1 Definition of RABA

We introduce a new distributed computing primitive, reproposable ABA (RABA). Unlike conventional ABA where replicas can vote once only, RABA allows replicas to change their votes.

Motivation. At a high level, RABA is designed to make the "naive attempt" mentioned in the introduction "work." Recall that in the naive approach, replica simply waits for the first $n - f$ RBCs to complete and propose 1 for the ABA instances whose corresponding RBC have completed, and propose 0 otherwise. This approach, however, may lead to a situation where no transactions may be delivered.

To fix the naive attempt, we aim at using RABA to replace ABA. A core theorem that we can prove is that if $n - f$ correct replicas deliver some proposals in $n - f$ RBC instances, then at least $f + 1$ correct replicas will propose 1 for at least $f + 1$ ABA instances. For these particular ABA instances, we would like to ensure that they will indeed decide 1. Thus, we require a property (called *biased validity*) that could imply the above fact: if $f + 1$ correct replicas *propose* 1, then any correct replica that terminates *decides* 1. Note that the property is different from the biased validity property defined in CKPS, as that definition was defined in the context of external validity.

Defining biased validity alone would not help achieve liveness. We need to make a syntactic change to ABA to make it *reproposable*: a replica who previously voted 0 (back then the corresponding RBC did not complete) can have a chance to vote 1 (if now the replica delivers the corresponding RBC). And we want to ensure RABA can terminate if all correct replicas either *propose* 1 or *repropose* 1. The property is called *biased termination*.

The above two properties are two core properties for RABA. Other properties are either the same as ABA or require some tweaks due to the syntactic difference between ABA and RABA.

RABA syntax (API). Syntactically, a RABA protocol tagged with a unique identifier *sid* is specified by $propose(sid, \cdot)$, $repropose(sid, \cdot)$, and $decide(sid, \cdot)$, where the input domain is $\{0, 1\}$. For our purpose, RABA is "biased towards 1." (Namely, "1" is the favorable value.) It is up to the high-level protocol to decide when to trigger the *propose* and *repropose* functions for each correct replica. Below are the function constraints enforced by correct replicas:

- $propose(sid, \cdot)$: Each replica can propose a value v at the beginning of the protocol *sid*. Each replica can propose a value only once.
- $repropose(sid, \cdot)$: A replica that proposed 0 is allowed to change its mind and repropose 1. A replica that proposed 1, however, is not allowed to repropose 0. If a replica reproposes 1, it does so at most once.
- $decide(sid, \cdot)$: A replica terminates the protocol identified by *sid* by generating a decide message.

Definitions of security. RABA (biased toward 1) satisfies the following properties:

- **Validity**: If all correct replicas *propose* v and never *repropose* $\bar{v}$, then any correct replica that terminates *decides* v.
- **Unanimous termination**: If all correct replicas *propose* v and never *repropose* $\bar{v}$, then all correct replicas eventually terminate.
- **Agreement**: If a correct replica *decides* v, then any correct replica that terminates *decides* v.
- **Biased validity**: If $f + 1$ correct replicas *propose* 1, then any correct replica that terminates *decides* 1.
- **Biased termination**: Let Q be the set of correct replicas. Let Q_1 be the set of correct replicas that propose 1 and never repropose 0. Let Q_2 be correct replicas that propose 0 and later repropose 1. If $Q_2 \neq \emptyset$ and $Q = Q_1 \cup Q_2$, then each correct replica eventually terminates.
- **Integrity**: No correct replica decides twice.

RABA has a slightly different validity property modified for our RABA syntax. It implies validity for two cases: 1) all correct replicas propose 1 (and, of course, they cannot repropose 0 according to our syntax) before termination; 2) all correct replicas propose 0 and never repropose 1 before termination.

Unanimous termination is weaker than the conventional termination property: it guarantees termination only when all correct replicas propose the same input. Validity and unanimous termination can be combined into a single property:

- If all correct replicas *propose* v and never *repropose* $\bar{v}$, then any correct replica *decides* v.

The agreement property of RABA is identical to that of ABA.

Biased validity in RABA requires that if $f + 1$ replicas, instead of all correct replicas, propose 1, then a correct replica that terminates decides 1. While the property echoes that of VABA [16], RABA is not in the context of "validated" ABA (VABA). This is precisely our goal, as VABA requires the usage of expensive (threshold) signatures which we strive to avoid. Nevertheless, this also means that when using RABA, we no longer have the powerful "validation" technique (and we need to be creative when using RABA).

Our biased termination property ensures that the protocol will eventually terminate as long as all correct replicas either propose 1 (and never repropose 0) or initially propose 0 but change their minds to repropose 1. Q_1 can be an empty set. Q_2 cannot be an empty set because otherwise biased termination is implied by unanimous termination.

Unanimous termination and biased termination complement each other. Remarkably, RABA does not have the usual termination property. (Correspondingly, our RABA protocol may indeed never terminate.) To use RABA in our favor, one must use a high-level protocol to control the inputs of RABA, allowing, when necessary, replicas to change their votes to attain termination eventually. Note we do not further restrict the APIs of RABA, as we find RABA in the current form is sufficient for our purpose. Restricting too much might impede novel usages of RABA.

Allowing a replica to change its mind and enabling a non-validated version of biased validity/termination properties are two central ideas underlying RABA.

One might wonder: why not use two ABA instances instead, one ABA for the "propose" phase and the other ABA for the "repropose" phase? One possible anomaly for this idea is that one replica might decide twice. Indeed, it is possible that while some replica is participating in the second ABA, the first ABA has terminated; it is also possible that some replicas terminate for the first ABA, while some other replicas terminate for the second ABA. According to our RABA definition, this is not allowed because the "integrity"—no correct replica decides twice—is introduced to govern both the propose operation and the repropose operation in a single RABA instance explicitly associated with a session identifier *sid*.

The overall formalization of RABA allows hiding—as much as *we could*—subtle protocol implementation details, exposing a clean API that can be neatly fit into a simple and novel asynchronous BFT framework—PACE.

4.2 Pisa: Efficient RABA from Pillar

We present a RABA protocol, Pisa, built on top of Pillar. As illustrated in Figure 3, we modify the round $r = 0$ of Pillar, while the code for round $r > 0$ remains unchanged. In particular, we make the following major changes in round 0. First, each replica p_i that proposes 0 can repropose 1. At ln 03-04, if the *repropose*() event is triggered, regardless of which round p_i is in, it broadcasts a $\text{bval}_0(1, \perp)$ message if it has not done so. Second, if a correct replica p_i proposes 1, it immediately adds 1 to bin_values_r (ln 08-09). If p_i has not previously broadcast an $\text{aux}_r()$ message, it broadcasts $\text{aux}_r(1, 1)$ (ln 10). Third, if p_i proposes 0 and receives $f + 1$ $\text{bval}_r(1, \perp)$ (ln 12-17), in addition to broadcasting $\text{bval}_r(1, \perp)$ (ln 14), it immediately adds 1 to bin_values_r (ln 16). If p_i has not sent any $\text{aux}_r()$ message, it also broadcasts $\text{aux}_r(1, 1)$ (ln 17). Last, ln 24 sets the value of

```
01 upon event $propose(sid, v_i)$
02   if $r = 0$, $est_0 \leftarrow v_i, maj_0 \leftarrow \perp$
03 upon event $repropose(id, 1)$
04   broadcast $\text{bval}_0(1, \perp)$                                {1st phase}
05 round 0
06   $bin\_values_r \leftarrow \emptyset$
07   broadcast $\text{bval}_r(est_r, maj_r)$
08   if $est_r = 1$                                         {biased toward 1}
09     $bin\_values_r \leftarrow \{1\}$
10     if $\text{aux}_r()$ has not been sent, broadcast $\text{aux}_r(1, 1)$
11   upon receiving $\text{bval}_r(*, *)$ from $j$
12     if there are $f + 1$ $\text{bval}_r(b, *)$
13       if $\text{bval}_r(b, maj_r)$ has not been sent
14         broadcast $\text{bval}_r(b, maj_r)$
15       if $b = 1$                   {enter the 2nd phase and biased toward 1}
16         $bin\_values_r \leftarrow bin\_values_r \cup \{b\}$
17         if $\text{aux}_r()$ has not been sent, broadcast $\text{aux}_r(1, 1)$
18     if there are $2f + 1$ $\text{bval}_r(b, *)$                    {2nd phase}
19       $bin\_values_r \leftarrow bin\_values_r \cup \{b\}$
20       if $\text{aux}_r()$ has not been sent, broadcast $\text{aux}_r(b, b)$
21   upon receiving $n - f$ $\text{aux}_r(*, *)$ where the sets of values carried by these
messages are $vals_r$ and $avals_r$; $vals_r$ and $avals_r$ are both subsets of $bin\_values_r$.
22     if $\text{V}_2(vals_r, b) \geq \lceil \frac{n+f+1}{2} \rceil$
23       $est_{r+1} \leftarrow b, maj_{r+1} \leftarrow b$
24       if $b = 1$, $decide(sid, b)$
25     else
26       $est_{r+1} \leftarrow 1, maj_{r+1} \leftarrow 1$
27     $r \leftarrow r + 1$, continue to the next round
```

Figure 3: The Pisa protocol for round 0 only at p_i.

common coin to 1 in round 0. Doing so ensures that if a replica receives $n - f$ $\text{aux}_r(1, 1)$, it will directly terminate the protocol. For each replica that receives both $\text{aux}_r(1, 1)$ and $\text{aux}_r(0, 0)$ (ln 26), the replica enters the next round using $est_{r+1} = 1$ and $maj_{r+1} = 1$.

Intuition and analysis. Our motivation for Pisa is that upon proposing 1, replicas directly broadcast both $\text{bval}_r(1, \perp)$ and $\text{aux}_r(1, 1)$ all at once, knowing that all correct replicas will either propose 1 or repropose 1. In other words, the protocol will eventually terminate and replicas can decide in only one step in the optimal case.

In round 0, a replica p_i puts 1 in bin_values_r and directly broadcasts $\text{aux}_r(1, 1)$ if its initial input is 1. This ensures that if more than $f + 1$ correct replicas have 1 as their input in round 0, no correct replicas can receive $2f + 1$ $\text{aux}_r(0, 0)$ and use 0 as input for the next round. As the common coin is set to 1 in round 0, all correct replicas will have 1 as the input for the next round, achieving the biased validity property.

Such an approach does not guarantee the conventional termination property of ABA. Consider a scenario with four replicas in Figure 4 (we use $\text{aux}_r(v)$ in the figure, as each replica broadcasts $\text{aux}_r(v, v)$ in round 0). Replica p_0 proposes 1 while p_1 and p_2 propose 0. The faulty replica p_3 can send $\text{bval}_r(0, \perp)$ to p_2 and p_3 and make them send $\text{aux}_r(0, 0)$. While p_0 can receive $n - f$ $\text{bval}_r(0, \perp)$ messages, put 0 to its bin_values_r, and accept $\text{aux}_r(0, 0)$, it has already sent $\text{aux}_r(1, 1)$. Since p_2 and p_3 can not receive enough $\text{bval}_r(1, \perp)$ messages, they will not put 1 to their bin_values_r. If p_3 does not send any $\text{aux}_r()$ messages, p_1 and p_2 are unable to move to the next round or terminate the protocol.

Pisa achieves biased termination if a reproposal is allowed. In particular, a replica is allowed to repropose 1 if it previously proposed 0. In this particular example, if correct replicas p_1 and p_2 repropose, they will send $\text{bval}_0(1, \perp)$. Since p_1 and p_2 are still in round 0, they are able to collect $n - f$ $\text{bval}_0(1, \perp)$ and put 1 to bin_values_r. Hence, all replicas will eventually move to the next round. As we use 1 as the common coin for round 0, correct replicas will eventually decide 1. Note that prior to the reproposal of p_1 and p_2, the faulty replica p_3 may have sent $\text{aux}_r(0, 0)$ to p_1 and p_2. In this case, both p_1 and p_2 receive 3 $\text{aux}_r(0, 0)$ messages and use 0 as input for round 1. Therefore, even if p_1 and p_2 repropose later, replicas already move to a new round such that the $\text{bval}_0(1, \perp)$ messages will not be accepted. That is, replicas might not necessarily decide 1 if fewer than $f + 1$ correct replicas propose 1.

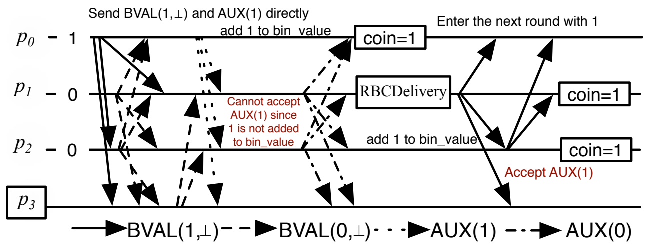


Figure 4: Example of biased termination of Pisa.

Note that Pisa can terminate with one step (round 0) in the best case. This is because if all correct replicas propose 1, they directly add 1 to bin_values_0, and broadcasts $\text{bval}_0(1, \perp)$ and $\text{aux}_0(1, 1)$ simultaneously. As the common coin value is 1 in round 0, all correct replicas decide in one step.

4.3 Converting ABA to RABA

Our strategy that converts ABA to RABA is generic. In fact, we find that we can apply the same measures to almost all (if not any) ABA protocols to RABA. At a high level, we only need to modify the first round of the protocol in a simple manner. First, the common coin value in the first round is fixed to 1. Second, upon the $propose(1)$ or $repropose(1)$ events, each correct replica can directly broadcast *all* the required messages (e.g., $\text{bval}_r(1)$, $\text{aux}_r(1)$) in the first round. Doing these ensure biased validity. We emphasize that due to the diversity of ABA protocols, our generic strategy is "informal." Thus, one needs to formally prove the resulting RABA protocols are indeed secure. We show in detail in Appendix G how to convert other protocols such as CKS [17], Crain's ABA protocols [25], and Cobalt [37] to RABA.

5 THE PACE FRAMEWORK AND THE PACE INSTANTIATIONS

We are now ready to describe PACE, our new asynchronous BFT framework. PACE uses *r-broadcast* and *r-deliver* primitives of RBC, and *propose*, *repropose* and *decide* primitives of RABA to overcome the two-subphase bottleneck. Figure 5 describes the pseudocode of our framework. For each epoch, the framework consists of n parallel RBC instances and n parallel RABA instances. In the RBC phase, each replica p_i r-broadcasts a proposal m_i for RBC_i. If p_i r-delivers a proposal from RBC_j, it proposes 1 for RABA_j. Upon delivery of $n - f$ RBC instances, instead of waiting for $n - f$ RABA instances to terminate, p_i immediately proposes 0 for all RABA instances that have not been started. If p_i later delivers a proposal from some RBC_j, it has proposed 0 for RABA_j, and has not terminated RABA_j, it reproposes 1 for RABA_j. Let S be the set of indexes that RABA_j

```
init
    $e \leftarrow 0$                                                {epoch number}
upon selecting $m_i$ from the buffer of $p_i$
    r-broadcast($[e, i], m_i$) for $\text{RBC}_i$
upon r-deliver($[e, j], m_j$) for $\text{RBC}_j$
    if $\text{RABA}_i$ has not been started
        propose($[e, j], 1$) for $\text{RABA}_j$
    else
        repropose($[e, j], 1$) for $\text{RABA}_j$
upon delivery of $n - f$ RBC instances
    for RABA instances that have not been started
        propose($[e, j], 0$)
upon decide($[e, j], v$) for any value $v$ for all RABA instances
    let $S$ be set of indexes for RABA instances that decide 1
    wait until r-deliver($[e, j], m_j$) for all $\text{RABA}_j$ such that $j \in S$
        a-deliver($\cup_{j \in S}\{m_j\}$) in some deterministic order
        $e \leftarrow e + 1$
```

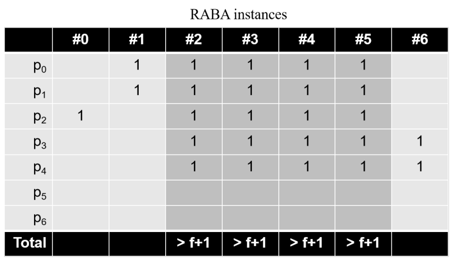
Figure 5: The PACE asynchronous BFT framework. The code for replica p_i.

decides 1. If RABA_j decides 1, the proposal from p_j is included in the final delivered set. After all RABA instances terminate and all RBC_i ($i \in S$) instances are delivered, p_i a-delivers $(\cup_{j \in S}\{m_j\})$.

RABA does not itself attain termination. We use RBC carefully to "control" the API of RABA and force RABA to meet the unanimous termination condition or the biased termination condition. To see this, we distinguish several cases:

- *Case 1: All correct replicas propose 1 for some RABA.* According to unanimous termination, the RABA instance eventually terminates with output 1.
- *Case 2: All correct replicas propose 0.* We further distinguish two cases:
    - *Case 2-1*: If they never repropose 1, the RABA instance eventually terminates due to unanimous termination.
    - *Case 2-2*: If some replicas repropose 1, then these replicas must have r-delivered the corresponding messages. Due to the agreement property of RBC, all correct replicas will deliver the messages and repropose 1. The protocol will terminate according to biased termination.
- *Case 3: Some correct replicas propose 0 and some other correct replicas propose 1.* Similar to Case 2-2, due to agreement of RBC, correct replicas will eventually repropose 1, and the RABA instance will terminate.

Thanks to the biased validity property, we can bound the number of transactions delivered for each epoch, conditioned on protocol termination. In particular, we prove that in the worst case (with a network scheduler), transactions from at least $f + 1$ replicas will be delivered. Indeed, according to the biased termination property, a RABA instance RABA_i will decide 1, if there are $f + 1$ or more correct replicas proposing 1. We observe that a correct replica will propose 1 for at least $2f + 1$ RABA instances (which is ensured by RBC). All correct replicas will input 1 for $(2f + 1)(2f + 1)$ inputs for all RABA instances. As there are at most $(3f + 1)(2f + 1)$ inputs for correct replicas, the total number of the 0 input from all correct replicas for all RABA instances is upper bounded by $(3f + 1)(2f + 1) - (2f + 1)(2f + 1) = 2f^2 + f$. Hence, the number of RABA instances that decide 0 is at most $\frac{2f^2+f}{f+1} < \frac{2f^2+2f}{f+1} = 2f$. That is, the number of RABA instances that decide 1 is at least $f + 1$. An example is shown in Figure 6 below.



RABA instances

	#0	#1	#2	#3	#4	#5	#6
p_0		1	1	1	1	1	
p_1		1	1	1	1	1	
p_2	1		1	1	1	1	
p_3			1	1	1	1	1
p_4			1	1	1	1	1
p_5							
p_6							
Total			> f+1	> f+1	> f+1	> f+1	

Figure 6: The number of RABA instances that decide 1 is at least $f + 1$. In this example, $f = 2$. Entries in each row i represent if a replica p_i input 1 for the corresponding RABA instances. Each replica proposes 1 for at least $2f + 1$ instances. (Each column represents a RABA instance and the inputs of replicas.) Regardless of how the 1's are placed (manipulated by a network scheduler), the number of RABA instances such that at least $f + 1$ correct replicas will propose 1 is at least $f + 1$. In this particular example, for RABA #2-#5, at least $f + 1$ correct replicas propose 1.

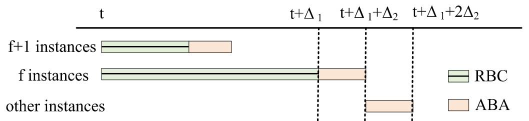
Note that in the worst case, BKR always delivers transactions from $n - f$ replicas in an epoch. This is not a benefit compared to PACE which delivers transactions from $f + 1$ replicas. BKR has to wait for $n - f$ transactions to proceed, causing transaction congestion. Also note that for PACE, in the normal case, transactions from at least $\lceil \frac{n+f+1}{2} \rceil$ replicas will be delivered. Jumping ahead, according to our experiments, in both failure and failure-free scenarios, the throughput of PACE is higher that of BKR.

In summary, while our paradigm does not improve the worst-case running time and communication complexity of BKR, it does improve the concrete time complexity. More importantly, it avoids the "two-subphase" bottleneck for BKR. Instead of waiting for at least $n - f$ ABA instances to terminate, each replica can now trigger all RABA instances once $n - f$ RBC instances are delivered. This improvement, not just triggering the ABA phase earlier, allows all RABA instances to run in a fully parallel manner, essentially avoiding transactions congestion.

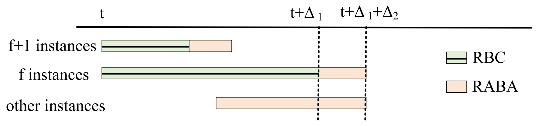
5.1 PACE vs. BKR: Worst-Case Scenario

We analyze the worst-case scenario of BKR and PACE. We let t be the time when an epoch begins, i.e., when any RBC is started. We let Δ_1 be the max delay an adversary could inject for all correct replicas to complete any RBC instance. We further let Δ_2 be the max delay an adversary could inject for any ABA protocol instance after every correct replica proposes some value. For RABA, we let Δ_3 be the max delay an adversary could inject on any RABA protocol after every correct replica proposes or reposposes some value that satisfies the predicate of biased termination. For simplicity, we let $\Delta_2 = \Delta_3$, as none of our RABA constructions introduce additional steps on top of ABA.

Figure 7a illustrates the worst-case scenario for the BKR diagram. An adversary could delay all the $2f + 1$ RBC instances (such that the sender is correct for any RBC) as much as possible until time $t + \Delta_1$. The f faulty replicas can also choose not to start their RBC instances. After the $2f + 1$ RBC instances complete, replicas will run the ABA protocols and complete them at the time no later than



(a) BKR.



(b) PACE.

Figure 7: PACE vs. BKR. Comparison of the worst-case scenario.

$t + \Delta_1 + \Delta_2$. Since the f RBC instances (such that the sender is faulty) are not even started, replicas will vote for 0 for the corresponding ABA instances. Thus, the entire epoch terminates at the time no later than $t + \Delta_1 + 2\Delta_2$.

In contrast, Figure 7b illustrates the situation for PACE. Each correct replica starts the RABA phase after it completes $2f + 1$ RBC instances. In the worst case, an adversary can simply delay some RABA instances (e.g., by triggering Case 3 mentioned for instances such that fewer than $f + 1$ correct replicas propose 1). Every correct replica eventually reproposes 1 according to the agreement property of RBC, i.e., no later than time $t + \Delta_1$, all correct replicas complete all RBC instances and propose or repropose for all RABA instances. Thus, the completion time of PACE is no later than $t + \Delta_1 + \Delta_2$. To conclude, in the worst case, PACE still outpaces BKR.

5.2 Efficient Instantiations

We instantiate our new framework using CrainH-R, CrainL-R, and Pisa as the underlying RABA protocols. The resulting protocols are called PACE-CrainH-R, PACE-CrainL-R, and PACE-Pisa, respectively.

Note that the correctness of our framework directly implies the correctness of all these protocols. It may still be helpful to examine an execution example. Here we discuss the same example in Figure 4. The biased termination condition for Pisa is met conditioned on RBC delivery in our framework. The agreement property of RBC ensures that if a correct replica r-delivers a request m, all correct replicas will eventually r-deliver m. Namely, if p_0 proposes 1, both p_1 and p_2 will eventually repropose 1. Thus, PACE-Pisa terminates due to biased termination.

6 IMPLEMENTATION AND EVALUATION

Implementation. We first implement six ABA protocols—Pillar, Pisa, CrainH, CrainH-R, CrainL, and CrainL-R. We use threshold PRF of Cachin, Kursawe, and Shoup [17] for common coins. For Pillar, Pisa, CrainL and CrainL-R, we use the $f + 1$ threshold. For CrainH and CrainH-R, we use the $2f + 1$ threshold. We use Pillar, CrainH, and CrainL in the BEAT library [2] and implement BEAT-Pillar, BEAT-CrainH, and BEAT-CrainL. We also instantiate the PACE framework using Pisa, CrainH-R, and CrainL-R, yielding PACE-Pisa, PACE-CrainH-R, and PACE-CrainL-R. We compare the six new protocols with BEAT (both BEAT-MMR and BEAT-Cobalt) and Dumbo [3]. Therefore, in total, we evaluate the performance of nine BFT protocols.

Overview. We deploy the protocols on Amazon EC2 utilizing up to 91 t2.medium VMs. Each VM has two vCPUs and 4GB memory. We evaluate both LAN and WAN settings, where the LAN VMs are launched in the same data center (DC), and the WAN VMs are evenly distributed in five continents. We evaluate the protocols using different number of replicas (i.e., network sizes) and batch sizes (i.e., contention levels). We use the number of the faulty replicas, f, to denote the network size. All transactions are of size 250 bytes. The LAN evaluation is limited to the case for $f = 1$, because our EC2 account in general cannot launch enough VMs in a single DC.

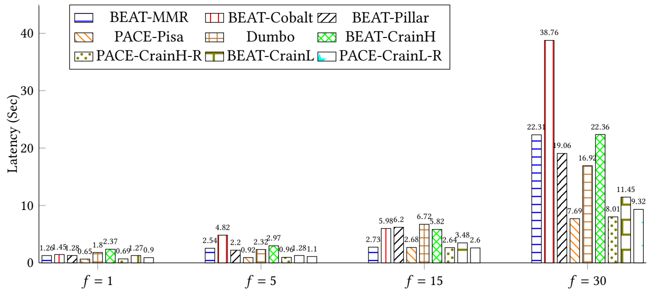


Figure 8: Latency of the protocols under no contention where the replicas are located in WANs. (This and subsequent figures are best viewed in color.)

Latency. We first evaluate the latency of the protocols under no contention in WANs, where each replica proposes only a batch of one transaction. We report the latency for $f = 1$, $f = 5$, $f = 15$, and $f = 30$. As illustrated in Figure 8, the latency of BEAT-Cobalt is consistently the highest among the protocols, mostly because Cobalt ABA has one more step in each round. Meanwhile, the latency of the PACE protocols (PACE-Pisa, PACE-CrainH-R, PACE-CrainL-R) is much lower than the others, because RABA terminates faster than ABA, and PACE has only one subphase. In contrast, the latency of Dumbo is in most of the cases slightly higher than the BKR protocols. The latency of PACE-Pisa is between 1/3 to 1/2 of the latency of Dumbo. We also evaluate the latency in the LAN for $f = 1$ (not shown in the figure): the latency of PACE-Pisa is 0.04s while the latency of Dumbo is 0.36s.

Throughput. In each epoch, each replica proposes B transactions. We simply let B be the batch size of transactions. Hence, all replicas propose in total nB transactions for an epoch. We evaluate the throughput and latency vs. throughput as B increases. We report the throughput of the BFT protocols for $f = 1$ in both LAN (Figure 9a) and WAN settings (Figure 9b) and report the throughput vs. latency in Figure 10a.

Protocols in the PACE framework (PACE-Pisa,PACE-CrainH-R, PACE-CrainL-R) outperform all other protocols. For instance, PACE-Pisa achieves 40% and 77% higher throughput than BEAT-Cobalt in the LAN and WAN settings, respectively. PACE-Pisa achieves 5.1x the throughput of Dumbo. This is due to the faster (biased) termination for RABA and the fully parallel feature of PACE-Pisa.

Furthermore, every PACE protocol outperforms its counterpart under the BKR framework (BEAT-Pillar, BEAT-CrainH, BEAT-CrainL). The throughput of PACE-Pisa, PACE-CrainH-R, and PACE-CrainL-R are 24.5%, 171%, and 15.6% higher than BEAT-Pillar, BEAT-CrainH, and BEAT-CrainL, respectively.

Among the three PACE protocols, PACE-Pisa achieves the best performance. PACE-CrainH-R achieves lower performance than PACE-Pisa, mainly due to the fact that replicas have to collect $2f + 1$ threshold PRF shares to generate common coins. PACE-CrainL-R achieves the lowest performance among the three, because each round of CrainL consists of more steps.

In blockchain applications, a typical block size is about 2MB, roughly matching a batch size $B = 2{,}000$ in our evaluation for $n = 4$ replicas. Looking at this setting, BEAT-Pillar and PACE-Pisa achieve 176% and 287% higher throughput than BEAT-Cobalt for WANs, respectively.

Scalability. We evaluate the throughput of the protocols by varying f from 2 to 30 in WAN. We report the throughput in Figure 9 and latency vs. throughput in Figure 10.

All the protocols under PACE framework outperform their counterparts under the BKR framework. Even the slowest PACE instantiation is more efficient than all other protocols.

For the three BEAT protocols (BEAT-Pillar, BEAT-CrainH, and BEAT-CrainL), BEAT-CrainL achieves the highest performance in most cases (except for $f = 1$). This may be due to the fact that CrainL has the good-case-coin-free property and has a fewer expected number of rounds than CrainH and Pisa (3 vs. 4).

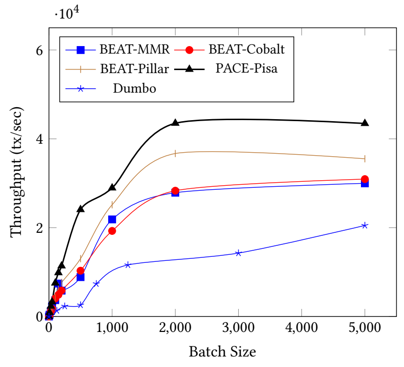
Different from the results we obtain for $f = 1$, as f grows, the performance difference among PACE-Pisa, PACE-CrainH-R, and PACE-CrainL-R become comparatively smaller. This may be due to the fact all these protocols have a fast and biased path (and it does not matter if the underlying ABA has a fast path). PACE-Pisa and PACE-CrainL-R perform better than PACE-CrainH-R. PACE-Pisa is also more efficient than PACE-CrainL-R, except for the case when n is medium-sized, where the two protocols offer interesting trade-offs.

For all the protocols, as the network size f increases, the throughput for these protocols first increases and then decreases. This echoes the results from prior works. When f grows, the number of transactions proposed concurrently grows accordingly. Nevertheless, when f further grows, the protocol itself becomes the bottleneck.

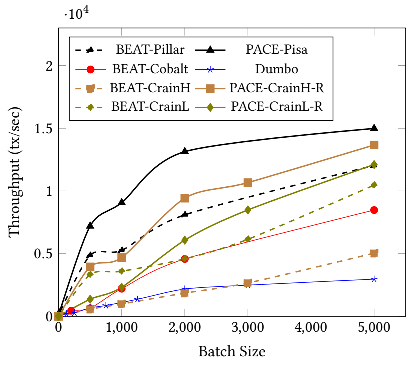
In the largest experiment ($n = 91$), the throughput of PACE-Pisa is around 15,994 tx/sec, around 3.6x the throughput of BEAT-Cobalt and 1.66x the throughput of Dumbo. (Recall for $f = 1$, PACE-Pisa achieves 1.77x the throughput of BEAT-Cobalt and 5.1x the throughput of Dumbo.)

Dumbo has lower throughput than BEAT-Cobalt for $f \leq 15$ and has roughly the same throughput as BEAT-Pillar for larger f's. BEAT-CrainH slightly outperforms Dumbo when $f \leq 15$ and Dumbo slightly outperforms BEAT-CrainL for $f = 20$ and $f = 30$.

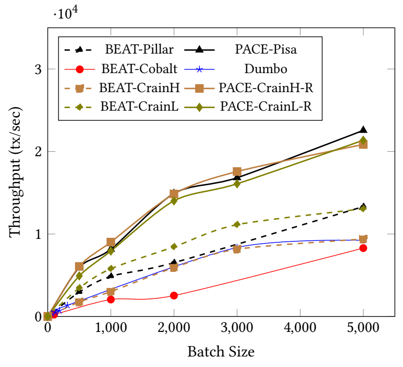
It is worth mentioning that the peak throughput in our scalability experiments can be higher if using larger batches. Indeed, prior works such as BEAT, EPIC, and Dumbo evaluated at least 10^4 and larger batch sizes (in our notation). We comment that evaluating a batch size larger than 5,000 for a network with a large f may be misleading. In all recent asynchronous BFT evaluations, to report the best possible throughput, each replica needs to propose disjoint



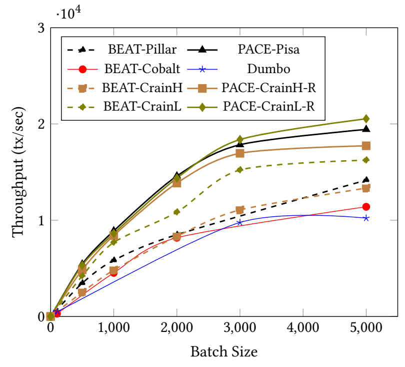
(a) Throughput for $f = 1$ where the replicas are located in the same DC.



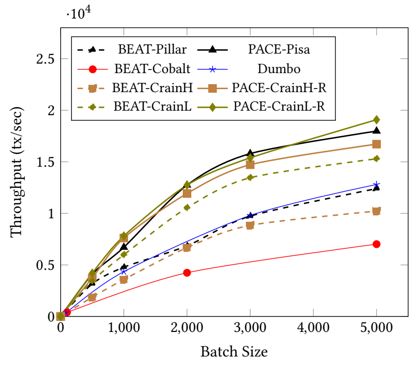
(b) Throughput for $f = 1$ where the replicas are from 4 different DCs.



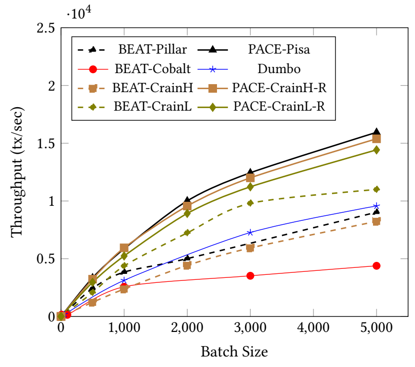
(c) Throughput for $f = 5$ where the replicas are from different DCs.



(d) Throughput for $f = 15$ where the replicas are from different DCs



(e) Throughput for $f = 20$ where the replicas are from different DCs.



(f) Throughput for $f = 30$ where the replicas are from different DCs.

Figure 9: Throughput of the protocols as f increases.

	BKR	NAIVE (insecure)	PACE-Pisa
$f = 1(n = 4)$	3.00	3.00	3.00
$f = 2(n = 7)$	5.54	5.45	5.66
$f = 5(n = 16)$	11.18	10.81	12.36
$f = 15(n = 46)$	31.24	28.12	33.62
$f = 20(n = 61)$	41.10	38.12	46.37
$f = 30(n = 91)$	61.00	54.75	65.25

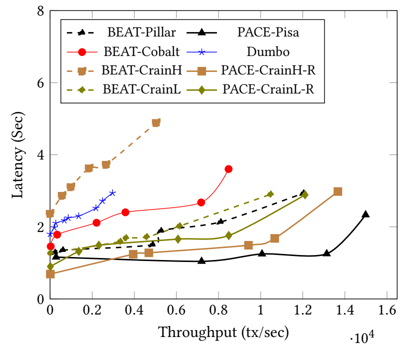
Table 3: The number of proposals that are delivered in ABA protocols for different frameworks. The number n is the total number of replicas and also the maximum number of proposals that could be delivered.

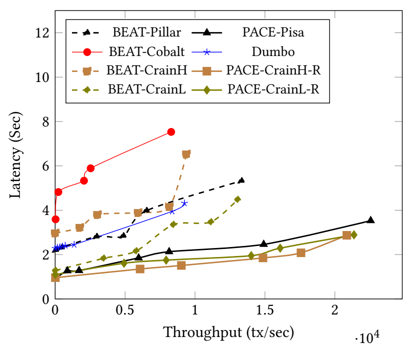
proposals. For $f = 30$ and $B = 5{,}000$, if each replica proposes a batch of around 1.2 MB transactions, all $n = 91$ replicas would propose over 100 MB transactions in total, where $n - f = 61$ batches are expected to include non-overlapping transactions. This would require each replica to have a huge buffer of pending transactions.

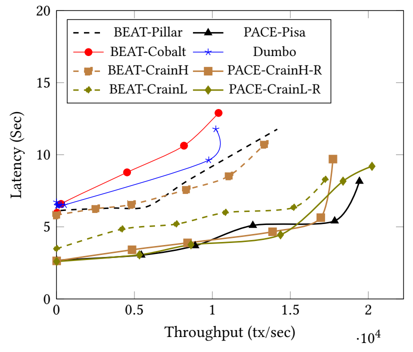
The number of proposals delivered in different BFT frameworks. We analyze the number of ABA or RABA instances that decide 1 to compare PACE and BKR frameworks. Just for performance comparison, we implement an incorrect framework directly running all instances in parallel (the "naive attempt"): after delivering $n - f$ RBC instances, a replica immediately starts the ABA instances that have not been started by proposing 0. We call it NAIVE here, and recall that in failure cases, its throughput can be 0 [11, 38].

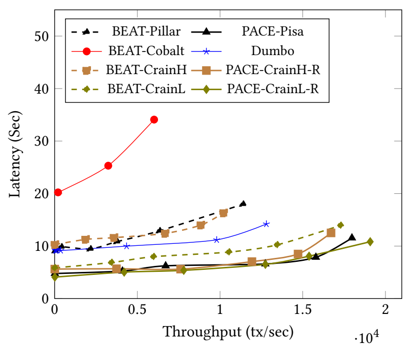
We summarize in Table 3 the number of ABA instances deciding 1, corresponding to the number of proposals delivered in failure-free scenarios for $f = 1$ in the WAN setting. We run the experiments 50 times for each network size and report the average number for all experiments. For almost all cases, the number of ABA instances that terminate with 1 in the BKR framework is close to $n - f$. In contrast, the number of ABA instances that terminate with 1 in NAIVE is visibly lower. This is because replicas do not wait for $n - f$ ABA instances to terminate with 1 before starting other ABA instances. Therefore, the number of delivered batches can be much lower than $n - f$. For our new framework, replicas tend to deliver 1 in ABA, and the number of ABA instances that terminate with 1 is shown to be slightly higher than that in BKR. Roughly, while our framework reduces the ABA phase latency (from two subphases to one), the efficiency of our framework is also higher.

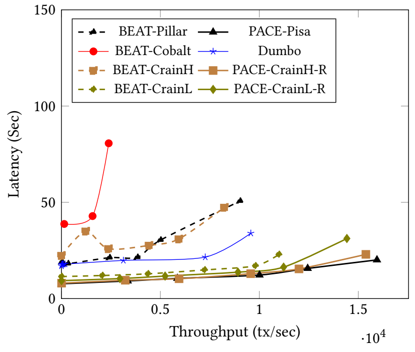
Performance under failures. We evaluate the performance of the protocols under failures by fixing $f = 5$ and $B = 5{,}000$. We evaluate three failure scenarios: 1) S_1-crash, where we let f replicas simply crash; 2) S_2-zero, where each faulty replica always broadcasts 0 in every step of ABA/RABA; 3) S_3-flip, where each replica replica


(a) Latency vs. throughput for $f = 1$.


(b) Latency vs. throughput for $f = 5$.


(c) Latency vs. throughput for $f = 15$.


(d) Latency vs. throughput for $f = 20$.


(e) Latency vs. throughput for $f = 30$.

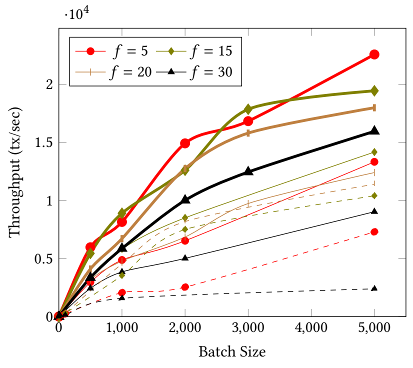

(f) Scalability where f varies from 2 to 30. Thick lines denote the throughput of PACE-Pisa, thin lines denote that of BEAT-Pillar, and dashed lines denote that of BEAT-Cobalt.

Figure 10: Scalability results where replicas are from different DCs.

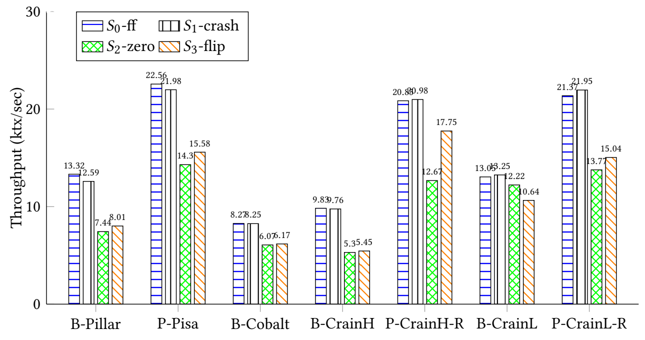


Figure 11: Performance of the protocols in failure scenarios in WANs for $f = 5$ and $B = 5{,}000$. We use B- and P- to denote BEAT- and PACE- respectively to save space in this figure.

always broadcasts a flipped value in every step of ABA/RABA. We compare the results with S_0-ff, the failure-free scenario.

The results are summarized in Figure 11. For all protocols, the performance under crash failures are similar to that in the failure-free scenario. When Byzantine failures occur, their performance degrade. We find that for most protocols under the BKR framework, the number of ABA instances that decide 1 is slightly larger than $n - f$, while the number of ABA instances that decide 1 under the PACE framework is between 8 to 12 (slightly lower than $n - f$). But this does not hurt the performance of PACE protocols: the performance degradation for PACE protocols, by percentage, is no larger than that for BEAT protocols. This is because PACE protocols do not have the two-subphase bottleneck and this is consistent with our analysis in Sec. 5.1.

We find that for all PACE protocols, the performance under all failure scenarios is still better than their individual counterparts in the BKR framework. Even more remarkably, among all failure scenarios, the slowest PACE instantiation (PACE-CrainH-R with S_2-zero) outpaces the most efficient BEAT instantiation in its failure-free scenario (BEAT-Pillar with S_0-ff).

7 CONCLUSION

We propose the notion of reproposable ABA (RABA) and use it to solve a long-standing problem of running ABA concurrently, leading to a fully parallelizable BFT framework (PACE) outperforming prior ones. We provide efficient instantiations of RABA and PACE. We show that all PACE instantiations outperform existing protocols in terms of all metrics in both failure-free and failure scenarios. All our instantiations rely on the well-established Diffie-Hellman assumptions and achieve standard 128-bit security.

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A REVIEW OF MMR AND COBALT ABA

The pseudocode of MMR is given in Figure 12. First, each replica broadcasts $\mathsf{bval}_r(est_r)$ where est_r is the input of the round (in round 0, est_r is the ABA vote). If a replica receives $f+1$ $\mathsf{bval}_r(v)$ and has not broadcast v, it broadcasts $\mathsf{bval}_r(v)$. Upon receiving $n-f$ $\mathsf{bval}_r(v)$, a replica adds v to bin_values_r. Then each replica sends an $\mathsf{aux}_r()$ message for the first value added to bin_values_r. Upon receiving $n-f$ $\mathsf{aux}_r()$ messages such that the set of values carried by these message, $vals$, is a subset of bin_values_r, the replica compares the value(s) with the common coin. If there is only one value in $vals$ messages and the value is the same as the coin, the replica decides it. Otherwise, the replica enters the next round and uses the common coin as est_r.

Cobalt ABA [37] has one additional $\mathsf{conf}_r()$ step in each round, also as shown in Figure 12.

B PSEUDOCODE OF THE BKR PROTOCOL

We show the pseudocode of the BKR paradigm in Figure 14 that uses the *r-broadcast* and *r-deliver* primitives of RBC, and *propose* and *decide* primitives of ABA.

```
upon event $propose(v_i)$
    if $r = 0$, $est_0 \leftarrow v_i$
round $r$
    $bin\_values_r \leftarrow \emptyset$
    broadcast $\text{bval}_r(est_r)$
    upon receiving $\text{bval}_r(v)$ from $f+1$ replicas
        if $\text{bval}_r(v)$ has not been sent, broadcast $\text{bval}_r(v)$
    upon receiving $\text{bval}_r(v)$ from $n-f$ replicas
        $bin\_values_r \leftarrow bin\_values_r \cup \{v\}$
    wait until $bin\_values_r \neq \emptyset$
        broadcast $\text{aux}_r(v)$ where $v \in bin\_values_r$
    upon receiving $n-f$ $\text{aux}_r()$ such that the set of values carried by these messages, $vals$, is a subset of $bin\_values_r$
        broadcast $\text{conf}_r(vals)$
    upon receiving $n-f$ $\text{conf}_r()$ such that the set of values carried by these messages, $vals$, is a subset of $bin\_values_r$
        $s \leftarrow \text{coin}_r$
        if $vals = \{b\}$
            $est_{r+1} \leftarrow b$
            if $b = s$, $decide(b)$
        else $est_{r+1} \leftarrow s$
    $r \leftarrow r+1$
```

Figure 12: MMR ABA and Cobalt ABA. Cobalt ABA has the boxed code, while MMR ABA does not have it.

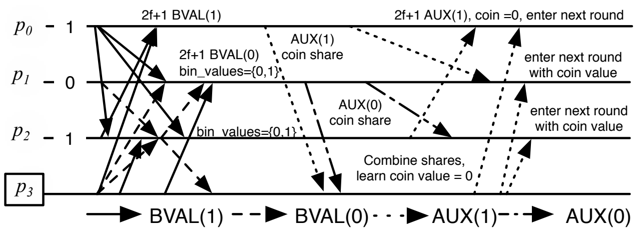
p_0 - 1
p_1 - 0
p_2 - 1
p_3
2f+1 BVAL(1)
2f+1 AUX(1), coin =0, enter next round
AUX(1) coin share
2f+1 BVAL(0) bin_values={0,1}
enter next round with coin value
AUX(0) coin share
bin_values={0,1}
enter next round with coin value
Combine shares, learn coin value = 0
BVAL(1)
BVAL(0)
AUX(1)
AUX(0)

Figure 13: The liveness issue of MMR. A faulty replica p_3 first sends $\text{bval}_r(1)$ to p_0. Since p_0 receives $\text{bval}_r(1)$ from p_2 and itself, p_0 will send $\text{aux}_r(1)$ and its threshold signature shares for the common coin. p_3 generates a share and combines the $f+1=2$ shares to obtain the common coin value (e.g., 0). p_3 then makes p_2 send $\text{aux}_r(1)$ by letting p_2 receive 3 $\text{bval}_r(1)$. Also, p_3 sends $\text{aux}_r(1)$ to p_0, making p_0 receive 3 $\text{aux}_r(1)$ (a value different from the common coin) and use 1 as the input for the next round. For p_1 and p_2, since they have added both 0 and 1 in their bin_values_r and receive both $\text{aux}_r(0)$ and $\text{aux}_r(1)$, they will enter the next round with the common coin value 0. In this way, the protocol may never terminate.

BKR proceeds in epochs initialized as $e = 0$. Each epoch e includes a RBC phase, including n parallel RBC instances RBC_i for $i \in [0..n-1]$, and an ABA phase, including n ABA instances ABA_i for $i \in [0..n-1]$, where RBC_i is triggered by replica $p_i \in [p_0..p_{n-1}]$ to r-broadcast a proposal m_i (a batch of transactions) selected from its buffer, and ABA_i is triggered by correct replicas to decide if m_i has been r-delivered. The ABA phase is not fully parallel, requiring that if a replica has not received some proposal from p_j during the RBC phase, the replica must abstain from proposing 0 for ABA_j until $n-f$ ABA instances decide 1. To isolate the primitive instances, we tag each message in the instances with e.

```
init
    $e \leftarrow 0$
upon selecting $m_i$ from the buffer of $p_i$
    $r\text{-}broadcast([e,i], m_i)$ for $\text{RBC}_i$
upon $r\text{-}deliver([e,j], m_j)$ for $\text{RBC}_j$
    if $\text{ABA}_j$ has not been started
        $propose([e,j], 1)$ for $\text{ABA}_j$
wait until termination of any $n-f$ ABA instances
    for all $\text{ABA}_j$ instances that have not been started
        $propose([e,j], 0)$
upon $decide([e,j], v)$ for any value $v$ for all ABA instances
    let $S$ be set of indexes for ABA instances that decide 1
    wait until $r\text{-}deliver([e,j], m_j)$ for all $\text{ABA}_j$ such that $j \in S$
        $a\text{-}deliver(\cup_{j \in S}\{m_j\})$ in some deterministic order
        $e \leftarrow e+1$
```

Figure 14: The BKR paradigm. The code for replica p_i for an epoch e.

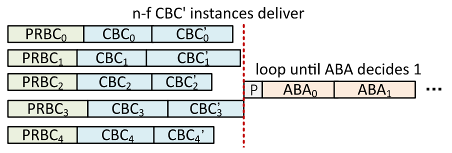

Figure 15: Dumbo. PRBC denotes provable reliable broadcast, a four-step protocol. CBC stands for consistent broadcast and has three steps [16]. Dumbo (built on top of CKPS) has one PRBC phase (4 steps), two CBC phases (2× 3 steps), one permutation step ("P"), and three sequential distribution (1 step) and ABA (3 for Cobalt ABA) instances. As each Cobalt ABA instance takes on average 3× 4 or 4× 4 steps, Dumbo has on average 50 or 62 steps.

C REVIEWING THE CKPS PARADIGM AND ITS DERIVATIVES

The CKPS paradigm reduces asynchronous BFT to (multi-valued) validated asynchronous Byzantine agreement (VABA) [16]. CKPS shows that VABA can be built using (verifiable) consistent broadcast (CBC) and ABA.

SINTRA [18]. SINTRA includes an atomic broadcast implementation of the CKPS paper. It uses Shoup's RSA threshold signature scheme and chooses to implement a simpler atomic broadcast protocol in CKPS that does not terminate in an expected constant number of rounds. Being the first CKPS instantiation, SINTRA is not optimized for high throughput.

Dumbo [31]. Dumbo includes two asynchronous BFT protocols—Dumbo1 and Dumbo2. Dumbo2 performs consistently better than Dumbo1, so we focus on Dumbo2 only (see Figure 15). Dumbo is motivated by the fact that the VABA construction in CKPS requires a large bandwidth. Instead of directly applying the bulk data to VABA, Dumbo introduces an additional provable RBC (PRBC) phase such that only the PRBC phase carries bulk data, and the VABA phase takes as input data fingerprints only. Hence, Dumbo has four more steps due to the PRBC phase, including three expensive n to n communication and an additional $O(n^3)$ pairing operations, incurring higher latency than CKPS. Our experiment shows BEAT-Pillar outpaces Dumbo until $n = 61$. Even when n is between 62 and 91, Dumbo has marginally higher throughput than BEAT-Pillar.

D PROOF OF CORRECTNESS FOR PILLAR

In this section, we prove the correctness of Pillar.

LEMMA D.1. *In round $r > 0$, if a correct replica p_i sets $\delta_r(v) = 1$ and another correct replica sets $\delta_r(\bar{v}) = 1$, $v = \bar{v}$.*

Proof. In round $r > 0$, correct replicas may send $\mathsf{bval}_r(*, v)$, $\mathsf{bval}_r(*, \bar{v})$, or $\mathsf{bval}_r(*, \perp)$ and do not change the maj_r value in the same round. There are two cases: $v = s_{r-1}$ and $v = \bar{s}_{r-1}$. We first show the case for $v = s_{r-1}$. Assume, towards a contradiction, there exists a replica p_i that receives $2f + 1$ $\mathsf{bval}_r(v, v)$ and $\mathsf{bval}_r(v, \perp)$. In our protocol, if p_i receives $2f + 1$ $\mathsf{bval}_r(v, v)$ and $\mathsf{bval}_r(v, \perp)$ and sets $\delta_r(v) = 1$, at least $f + 1$ correct replicas have sent $\mathsf{bval}_r(v, v)$ or $\mathsf{bval}_r(v, \perp)$, but will never send $\mathsf{bval}_r(v, \bar{v})$ or $\mathsf{bval}_r(\bar{v}, \bar{v})$. On the other hand, if p_j sets $\delta_r(\bar{v}) = 1$, it has received $2f + 1$ $\mathsf{bval}_r(\bar{v}, \bar{v})$, at least $f + 1$ of which are sent by correct replicas. Therefore, at least one correct replica has sent both $\mathsf{bval}_r(v, v)$ (or $\mathsf{bval}_r(v, \perp)$) and $\mathsf{bval}_r(\bar{v}, \bar{v})$, which is a contradiction. The case for $v = \bar{s}_{r-1}$ can be proved similarly. If p_i sets $\delta_r(v) = 1$, it receives $2f+1$ $\mathsf{bval}_r(v, v)$. If p_j sets $\delta_r(\bar{v}) = 1$, it receives $2f + 1$ $\mathsf{bval}_r(\bar{v}, \bar{v})$ or $\mathsf{bval}_r(\bar{v}, \perp)$. In other words, at least one correct replica has sent both $\mathsf{bval}_r(*, v)$ and $\mathsf{bval}_r(*, \bar{v})$ (or $\mathsf{bval}_r(*, \perp)$), which is impossible. □

LEMMA D.2. *In round $r > 0$, if all correct replicas have the same input v, any correct replica either enters round $r + 1$ with v, or decides v in round r .*

Proof. In round $r > 0$, correct replicas may send $\mathsf{bval}_r(v, v)$, $\mathsf{bval}_r(v, \bar{v})$, or $\mathsf{bval}_r(v, \perp)$. Hence, correct replicas will not receive more than $f + 1$ $\mathsf{bval}_r(\bar{v}, *)$ and no correct replica will put $\bar{v}$ in its bin_values_r. Therefore, all correct replicas will send $\mathsf{aux}_r(v, v)$ or $\mathsf{aux}_r(\perp, v)$ and will not accept $\mathsf{aux}_r(\bar{v}, *)$ or $\mathsf{aux}_r(*, \bar{v})$. Each correct replica therefore eventually receives either $2f + 1$ $\mathsf{aux}_r(v, *)$ such that $\mathsf{V}_2(vals_r, v) \geq 2f + 1$ or $2f + 1$ $\mathsf{aux}_r(*, v)$ such that $\mathsf{V}_2(avals_r, v) \geq 2f + 1$.

If a replica p_i receives $2f + 1$ $\mathsf{aux}_r(v, v)$, it either decides v (for the case $v = s_r$), or enters the next round with v as est_{r+1} (for the case $v \neq s_r$). If p_i receives both $\mathsf{aux}_r(v, v)$ and $\mathsf{aux}_r(\perp, v)$ (or only $\mathsf{aux}_r(\perp, v)$), $\mathsf{V}_2(avals_r, v) \geq 2f + 1$ is satisfied. In this case, p_i will either decide v (for the case $v = s_r = s_{r-1}$), or enter the next round with v as est_{r+1} (for the case $v \neq s_r$ or $v \neq s_{r-1}$). □

THEOREM D.3. ***(Validity)*** *If all correct replicas propose v, then any correct replica that terminates decides v.*

PROOF. The proof follows from the following two lemmas.

LEMMA D.4. *In round $r > 0$, if all correct replicas have the same input v, any correct replica that terminates decide v.*

Proof. Lemma D.2 shows that correct replicas will either enter round $r + 1$ with v or decide v. If correct replicas start round r with the same input v and enter the next round, the input for round $r + 1$ must be v. As the probability that the common coin value equals v is 1/2, the probability that the protocol terminates in round $r + 1$ is 1/2. It is straightforward to see that any replica that terminates decides v. □

LEMMA D.5. *If all correct replicas propose v in round 0, any correct replica that terminates decides v.*

Proof. We show that if all correct replicas propose est_0 in round 0, correct replicas either terminate in the current round with est_0 or enter round 1 with the same est_r. As proven in Lemma D.2 and Lemma D.4, any correct replica that terminates decides v.

In round $r = 0$, all correct replicas broadcast $\mathsf{bval}_r(v, \perp)$. Since all correct replicas have the same input v and there are only f faulty replicas, correct replicas will not receive more than $f+1$ $\mathsf{bval}_r(\bar{v}, \perp)$ or send $\mathsf{bval}_r(\bar{v}, \perp)$. All correct replicas will eventually receive $2f+1$ $\mathsf{bval}_r(v, \perp)$ and send $\mathsf{aux}_r(v, v)$. Since no correct replica puts $\bar{v}$ in bin_values_r, all correct replicas will have only one single value v. If a correct replica terminates in round 0 and $v = s_r$, it decides v. If $v \neq s_r$, correct replicas use v as the input est_r for round 1. According to Lemma D.4, any correct replica that terminates decides v. □

This completes of the proof of the theorem. ■

THEOREM D.6. ***(Agreement)*** *If a correct replica decides v, then any correct replica that terminates decides v.*

PROOF. We prove agreement by showing that if p_i decides v in round r, all other correct replicas either decide in the same round or enter the next round with v as est_r. Assume, towards a contradiction, that another correct replica p_j enters the next round with $\bar{v}$.

LEMMA D.7. *If p_i terminates in round r and decides v, any correct replica p_j either 1) terminates in round r and decides v; or 2) enters round $r + 1$ with v as est_{r+1}.*

Proof. If p_i decides v in round r, there are three cases: 1) p_i receives at least $2f + 1$ $\mathsf{aux}_r(v, v)$ messages and $v = s_r$; 2) p_i receives both $\mathsf{aux}_r(v, v)$ and $\mathsf{aux}_r(\perp, *)$. Also, p_i receives at least a quorum of $\mathsf{aux}_r(*, v)$ messages, $v = s_{r-1}$, and $v = s_r$; For p_j, if it enters the next round with value $est_{r+1} = \bar{v}$, it cannot use the common coin value as input, as v is the common coin. Therefore, one the following conditions must apply: A) p_j receives at least $2f + 1$ $\mathsf{aux}_r(\bar{v}, \bar{v})$; B) p_j receives both $\mathsf{aux}_r(\bar{v}, \bar{v})$ and $\mathsf{aux}_r(\perp, \bar{v})$. At least a quorum of the messages are of the form $\mathsf{aux}_r(*, \bar{v})$; C) p_j receives both $\mathsf{aux}_r(\bar{v}, *)$ and $\mathsf{aux}_r(\perp, *)$ and $\bar{v} = s_{r-1}$. We now distinguish the two cases for p_i and show that none of the three conditions for p_j can be satisfies.

Case 1: p_i receives at least $2+1$ $\mathsf{aux}_r(v, v)$. At least $f+1$ correct replicas have sent $\mathsf{aux}_r(v, v)$. 1) If condition A is true, p_j receives $2f + 1$ $\mathsf{aux}_r(\bar{v}, \bar{v})$, at least $f+1$ of which are sent by correct replicas. Therefore, at least one correct replica has sent both $\mathsf{aux}_r(v, v)$ to p_i and $\mathsf{aux}_r(\bar{v}, \bar{v})$ to p_j, which is impossible. Condition A cannot be true. 2) If condition B is true, p_j receives a quorum of $\mathsf{aux}_r(*, \bar{v})$ messages, $f + 1$ of which are sent by correct replicas. Therefore, at least one correct replica has sent $\mathsf{aux}_r(v, v)$ to p_i and sent $\mathsf{aux}_r(*, \bar{v})$ to p_j, which is impossible. Condition B cannot be true. 3) If condition C is true, p_j receives only $\mathsf{aux}_r(\bar{v}, *)$ and $\mathsf{aux}_r(\perp, *)$. In other words, at least one correct replica has sent $\mathsf{aux}_r(v, v)$ to p_i and $\mathsf{aux}_r(\bar{v}, *)$ (or $\mathsf{aux}_r(\perp, *)$) to p_j, which is impossible. Condition C cannot be true.

Case 2: p_i receives both $\mathsf{aux}_r(v, v)$ *and* $\mathsf{aux}_r(\perp, *)$. *Also, p_i receives at least a quorum of* $\mathsf{aux}_r(*, v)$ *messages, $v = s_{r-1}$, and $v = s_r$.* 1) If condition A is true, p_j receives at least $2f+1$ $\mathsf{aux}_r(\bar{v}, \bar{v})$, at least $f+1$ are sent by correct replicas. Also, p_i receives $2f + 1$ $\mathsf{aux}_r(v, v)$ and $2f + 1$ $\mathsf{aux}_r(\perp, *)$. Therefore, at least one correct replica must have sent $\mathsf{aux}_r(\bar{v}, \bar{v})$ to p_j and $\mathsf{aux}_r(v, v)$ (or $\mathsf{aux}_r(\perp, *)$) to p_i, which is impossible. Condition A cannot be true. 2) If condition B is true, p_j receives both $\mathsf{aux}_r(\bar{v}, \bar{v})$ and $\mathsf{aux}_r(\perp, \bar{v})$, and at least a quorum of the messages are of the form $\mathsf{aux}_r(*, \bar{v})$. Since p_i receives a quorum of $\mathsf{aux}_r(*, v)$ messages, at least one correct replica has sent $\mathsf{aux}_r(*, v)$ to p_i and $\mathsf{aux}_r(*, \bar{v})$ to p_j. Condition B cannot be true. 3) p_j only receives $\mathsf{aux}_r(\bot, *)$ and $\mathsf{aux}_r(\bar{v}, *)$ and $\bar{v} = s_{r-1}$. This cannot be true since $v = s_{r-1}$. □

For the two cases, it is clear that if p_j decides in round r, it outputs v. We now show that if p_j terminates in round r' and decides $\bar{v}$, $v = \bar{v}$ where $r' > r$. In round r', one of the following cases must apply: A) p_j receives at least $2f + 1$ $\mathsf{aux}_{r'}(\bar{v}, \bar{v})$; B) p_j receives both $\mathsf{aux}_{r'}(\bar{v}, \bar{v})$ and $\mathsf{aux}_{r'}(\bot, \bar{v})$. Also, $\bar{v} = s_{r'-1}$ and $\bar{v} = s_{r'}$. If any of the two cases is true, at least one correct replica has sent $\mathsf{bval}_{r'}(\bar{v}, *)$. Meanwhile, according to Lemma D.7, any correct replica that enters round $r+1$ uses v as input. Furthermore, according to Lemma D.2, if all correct replicas enter round $r+1$ and use v as input, any correct replica either decides v or uses $est_{r+2} = v$ and enters round $r+2$. Therefore, there exists some round r'' where $r \leq r'' \leq r'$, a correct replica uses $\bar{v}$ as input and broadcasts $\mathsf{bval}_{r''}(\bar{v}, *)$, i.e., Lemma D.2 is violated, a contradiction. ■

Theorem D.8. ***(Termination)*** *All correct replicas eventually terminate the protocol.*

Proof. The proof is divided in two parts: 1) In each round, each correct replica will eventually proceed to the next round, and 2) a correct replica terminates the protocol with probability 1/2.

We prove the first part. In our protocol, est_r is always a binary value, either 0 or 1. A correct replica may send $\mathsf{bval}_r(v, *)$ or $\mathsf{bval}_r(\bar{v}, *)$. Also, there are at least $2f + 1$ correct replicas, among which at least $f + 1$ correct replicas propose the same value. Therefore, if $\bar{v}$ is different from its proposed value, each correct replica p_i will forward $\mathsf{bval}_r(\bar{v}, maj_r)$. Correct replicas will eventually receive $2f + 1$ $\mathsf{bval}_r()$ messages for at least one binary value (e.g., v), and send $\mathsf{aux}_r(v, *)$ or $\mathsf{aux}_r(\bot, *)$. Similarly, all correct replicas eventually receive $2f + 1$ $\mathsf{aux}_r()$ messages and proceed to the next round.

We now prove the second part. In particular, we prove in round r, if a correct replica p_i enters round $r + 1$ with $est_{r+1} = \bar{v}$, the protocol will terminate with 1/2 probability. (Hence, the $\bar{v}$ cannot be manipulated by the adversary such that $\bar{v}$ is always different from the common coin.)

If p_i enters the next round with $est_{r+1} = v = \bar{s}_r$, it must satisfy one of the three conditions: 1) p_i receives at least $2f + 1$ $\mathsf{aux}_r(v, v)$ messages; 2) p_i receives both $\mathsf{aux}_r(v, v)$ and $\mathsf{aux}_r(\bot, *)$. At least a quorum of the messages are $\mathsf{aux}_r(*, v)$; 3) p_i receives both $\mathsf{aux}_r(v, v)$ and $\mathsf{aux}_r(\bot, *)$ and $v = s_{r-1}$.

For the first case, at least $f+1$ correct replicas have sent $\mathsf{aux}_r(v, v)$. According to the Lemma D.1, we know that in each round, correct replicas send $\mathsf{aux}_r(v, v)$ for at most one value v. In other words, if at least one correct replica sends $\mathsf{aux}_r(v, v)$, no correct replica sends $\mathsf{aux}_r(\bar{v}, \bar{v})$. Hence, each correct replica receives at least one $\mathsf{aux}_r(v, v)$. According to Lemma D.1, correct replicas may send $\mathsf{aux}_r(v, v)$, $\mathsf{aux}_r(\bot, v)$, $\mathsf{aux}_r(\bot, \bar{v})$, but will not send $\mathsf{aux}_r(\bar{v}, *)$. No correct replica will set $\delta_r(\bar{v}) = 1$. The value v is determined based on the $majs$ values by the replicas so it cannot be manipulated by the adversary. Therefore, with a probability of 1/2, p_i decides v. Otherwise p_i enter the next round and use $est_{r+1} = v$.

For the second case, p_i decides if $b = s_{r-1}$ and $b = s_r$. With a probability of 1/2, $s_{r-1} = s_r$.

For the third case, with a probability of 1/2, $s_{r-1} = s_r$. In this case p_i will not decide. p_i uses the value of s_r as est_{r+1} with a probability of 1/2. ■

Theorem D.9. ***(Integrity)*** *No correct replica decides twice.*

Proof. In each round, a replica will only sends $\mathsf{aux}_r()$ message once and accepts only one $\mathsf{aux}_r()$ message from each replica. If a replica p_i decides v in round r, it has received $2f + 1$ $\mathsf{aux}_r(v, *)$ messages with the same v, or received $2f+1$ $\mathsf{aux}_r(*, v)$. If p_i decides twice, it must have received $2f + 1$ $\mathsf{aux}_r(\bar{v}, \bar{v})$, or received $2f + 1$ $\mathsf{aux}_r(*, v)$. Neither case is possible. Integrity thus follows. ■

E PROOF OF CORRECTNESS FOR PISA

In this section, we prove the correctness of Pisa.

Theorem E.1. ***(Validity)*** *If all correct replicas propose v and never repropose $\bar{v}$, then any correct replica that terminates decides v.*

Proof. If all correct replicas propose v and do not repropose $\bar{v}$, all correct replicas will only have v in their bin_values_r. Hence, replicas do not accept an $\mathsf{aux}_r(\bar{v}, \bar{v})$ message. Each correct replica will collect $2f + 1$ $\mathsf{aux}_r(v, v)$. If $v = 1$, replicas decide in round 0. Otherwise replicas enter round 1. Starting from round 1, Pisa follows Pillar. Therefore, according to Lemma D.2 and Lemma D.4, any correct replica that terminates decides v. ■

Theorem E.2. ***(Unanimous termination)*** *If all correct replicas propose v and never repropose $\bar{v}$, then all correct replicas eventually terminate.*

Proof. If all correct replicas propose v, they will all send $\mathsf{bval}_r(v, \bot)$. All correct replicas eventually put v to bin_values_r. Correct replicas will not send or accept any $\mathsf{aux}_r(\bar{v}, \bar{v})$ message and will accept either $\mathsf{aux}_r(v, v)$ or $\mathsf{aux}_r(\bot, v)$. According to Lemma D.2, all correct replicas will enter the next round with the same est_{r+1} value (including the case where replicas decide). According to the property of common coin, we know that correct replicas decide in each round with probability 1/2. Hence, all correct replicas eventually terminate. ■

Theorem E.3. ***(Agreement)*** *If a correct replica decides v, then any correct replica that terminates decides v.*

Proof. We first show the case where a correct replica p_i decides v in round $r = 0$. First of all, a correct replica p_j cannot decide $\bar{v}$ in round 0. This is because a correct replica decides a value only when the value equals the common coin, which is 1 in round 0. We now consider the case where p_i still decides in round 0 and p_j decides in round $r > 0$. We prove the following lemma:

Lemma E.4. *If p_i decides in round 0, any correct replica either decides in round 0 or uses 1 as input for round 1.*

Proof. If p_i decides in round 0, it decides $v = 1$ (the common coin value in round 0 is 1). Assume, towards a contradiction, that a correct replica p_j enters round 1 with 0 as input. In this case, the function $\mathsf{V}_2(vals_r, 0) \geq 2f + 1$ must be true. Hence, excluding the $\mathsf{aux}_r(\bot, *)$ messages, the number of $\mathsf{aux}_r(0, 0)$ messages p_j receives is greater than $2f+1$. Among the replicas that sent $2f+1$ $\mathsf{aux}_r(v,v)$ and $\mathsf{aux}_r(\bar{v},\bar{v})$ messages, at least one correct replica must have sent both $\mathsf{aux}_r(v,v)$ and $\mathsf{aux}_r(\bar{v},\bar{v})$. This is a contradiction, since a correct replica broadcasts $\mathsf{aux}_r()$ once in each round. □

Starting from round $r=1$, Pisa is the same as Pillar. Therefore, according to Lemma D.2 and Lemma D.4, any correct replica that terminates decides $v=1$.

For the case where p_i decides in round $r>0$, agreement simply follows that of Pillar, as Pisa is the same as Pillar starting from $r>0$. ■

Theorem E.5. ***(Biased validity)*** *If $f+1$ correct replicas propose 1, then any correct replica that terminates decides 1.*

Proof. In round 0, correct replicas will directly send $\mathsf{aux}_r(1,1)$ and will not send $\mathsf{aux}_r(0,0)$. Therefore, all correct replicas will either receive $2f+1$ $\mathsf{aux}_r(1,1)$ or both $\mathsf{aux}_r(1,1)$ and $\mathsf{aux}_r(0,0)$, but not $2f+1$ $\mathsf{aux}_r(0,0)$. This is because if a correct replica receives $2f+1$ $\mathsf{aux}_r(0,0)$, at least $f+1$ correct replicas must have sent $\mathsf{aux}_r(0,0)$. Therefore, at least one correct replica must have sent both $\mathsf{aux}_r(0,0)$ and $\mathsf{aux}_r(1,1)$, which is impossible. If a correct replica receives $2f+1$ $\mathsf{aux}_r(1,1)$, it directly decides. Otherwise it uses the common coin value 1 to enter the next round. Since Pisa is the same as Pillar starting from round 1, according to the Lemma D.2 and Lemma D.4, all correct replicas that terminate decide 1. ■

Theorem E.6. ***(Biased termination)*** *Let Q be the set of correct replicas. Let Q_1 be the set of correct replicas that propose 1 and never repropose 0. Let Q_2 be correct replicas that propose 0 and later repropose 1. If $Q_2 \neq \emptyset$ and $Q = Q_1 \cup Q_2$, then each correct replica eventually terminates.*

Proof. The proof consists of two parts: round $r=0$ and round $r>0$. We first prove the first case ($r=0$) that a correct replica either decides in round 0 or moves to round 1. Depending on the proposed values of replicas, there are three cases: 1) at least $f+1$ correct replicas propose 1; 2) at least one but fewer than $f+1$ correct replicas propose 1; 3) all correct replicas propose 0. We show that each replica can collect $2f+1$ $\mathsf{aux}_r()$ messages.

Case 1: More than $f+1$ correct replicas propose 1. All correct replicas will eventually receive $f+1$ $\mathsf{bval}_r(1,\perp)$. According to the protocol, all correct replicas will eventually receive $2f+1$ $\mathsf{bval}_r(1,\perp)$, put 1 in *bin_values*$_r$, and accept $\mathsf{aux}_r(1,1)$. Correct replicas may send $\mathsf{aux}_r(1,1)$ or $\mathsf{aux}_r(0,0)$. If a correct replica sends $\mathsf{aux}_r(0,0)$, it previously received $2f+1$ $\mathsf{bval}_r(0,\perp)$ messages, among which at least $f+1$ replicas are correct. Therefore, all correct replicas will eventually put 0 in their *bin_values*$_r$ and accept $\mathsf{aux}_r(0,0)$. All correct replicas can then decide or move to round 1.

Case 2: At least one but fewer than $f+1$ correct replicas propose 1. In this case $|Q_1| < f+1$ and $|Q_2| \geq f+1$. This case implies that at least $f+1$ correct replicas propose 0. In this case, all correct replicas will eventually receive $2f+1$ $\mathsf{bval}_r(0,\perp)$ and put 0 to *bin_values*$_r$. It is, however, not guaranteed that a correct replica will receive $2f+1$ $\mathsf{bval}_r(1,\perp)$ and put 1 in *bin_values*$_r$. Therefore, some correct replica that only has 0 in *bin_values*$_r$ will not accept an $\mathsf{aux}_r(1,1)$ message. The termination is guaranteed by the fact that correct replicas in Q_2 repropose 1. In particular, correct replicas will repropose 1, making correct replicas eventually receive a quorum of $\mathsf{bval}_0(1,\perp)$ messages. Hence, correct replicas will either enter the next round or eventually collect a quorum of $\mathsf{bval}_0(1,\perp)$ messages. In the latter case, correct replicas will be able to accept both $\mathsf{aux}_r(0,1)$ and $\mathsf{aux}_r(1,1)$ in round 0. Hence, correct replicas will either terminate in round 0 or move to the next round.

Case 3: All correct replicas have 0 as their input. In this case $|Q_1|=0$. All correct replicas will receive a quorum of $\mathsf{bval}_r(0,\perp)$ messages and add 0 to *bin_values*$_r$. Furthermore, each replica in Q_2 may repropose. Therefore, all correct replicas will receive $2f+1$ $\mathsf{bval}_r(1,\perp)$ and put 1 to *bin_values*$_r$. It is easy to see that all correct replicas will either decide in round 0 or move to the next round.

For the case where round $r>0$, since Pisa is the same as Pillar, agreement follows that of Pillar. This completes the proof of the theorem. ■

Theorem E.7. ***(Integrity)*** *No correct replica decides twice.*

Proof. In each round, each replica will only send $\mathsf{aux}_r()$ message once and accept one $\mathsf{aux}_r()$ message from each replica. Hence, only one value will be decided and integrity easily follows. ■

F PROOF OF CORRECTNESS FOR OUR BFT FRAMEWORK

We prove the correctness of our BFT framework. The proof immediately implies the correctness of our BFT instantiations. In particular, we will prove the following properties that are equivalent to the definitions of security for BFT:

- **Set agreement**: If any correct replica outputs a set V, then each correct replica outputs V.
- **Efficiency (validity)**: If a correct replica outputs a set V, then V contains a proposal from at least one correct replica.
- **Liveness**: If a proposal is submitted to all correct replicas, then all correct replicas eventually output a set containing some value.

Note that the above definitions generalize and relax prior definitions for systems on asynchronous common subset (ACS) such as HoneyBadgerBFT, BEAT, and Dumbo, where they all consider the following efficiency property:

- **Efficiency (validity) for some prior ACS definitions**: If a correct replica outputs a set V, then V contains proposals from at least $n-2f$ correct replicas.

The original idea of ACS in BKR requires replicas to agree on a common subset but does not restrict the size of the set V. Our efficiency definition thus echoes that of the original BKR paper, requiring V contains at least one proposal from one correct replica. This is—not at all—a "drawback." First, asking V to contain proposals from $n-2f$ correct replicas is unnecessary; our relaxed definitions are equivalent to the standard definitions for BFT. Second, the efficiency property defined in the previous work may restrict novel or efficient constructions: a system slowly delivering more transactions may not be more efficient than a system delivering fewer transactions but delivering them faster. Third, one can *easily* construct a system that satisfies the efficiency property requiring to output a set containing at least $n-2f$ replicas using a system with our efficiency property.

We begin with the following lemma.

Lemma F.1. *If all correct replicas are activated on some proposals for epoch e, then all correct replicas eventually terminate for epoch e.*

Proof. If all correct replicas are activated on some proposals for some epoch e, they will r-broadcast their proposals. Eventually, all correct replicas will r-deliver at least $2f + 1$ RBC instances. Hence, all correct replicas will proposes 0 for all RABA instances that have not been started. We distinguish all possible cases and show for each case all RABA instances will terminate.

We first consider case 1, where all correct replicas propose 1 for a RABA. In this case, according to unanimous termination, the RABA instance eventually terminates.

We now consider case 2, where all correct replicas propose 0. In this case, we further distinguish two sub-cases: 1) If they never repropose 1, the RABA instance eventually terminates due to unanimous termination. 2) If some replicas repropose 1, then these replicas must have r-delivered the corresponding messages. According to the agreement property of RBC, all correct replicas will deliver the messages and repropose 1. The protocol will terminate due to biased termination.

Finally, we consider case 3, where some correct replicas propose 0 and some other correct replicas propose 1. The case is similar to Case 2-2. Due to the agreement property of RBC, correct replicas will eventually repropose 1, and the RABA instance will terminate.

Therefore, the protocol will eventually terminate. □

We now prove set agreement.

Theorem F.2. ***(Agreement)** If any correct replica outputs a set V of proposals from replicas, then each correct replica outputs the same set V.*

Proof. We consider an epoch e, where a correct replica p_j a-delivers a set V. According to our protocol, the set V is a set of proposals from different replicas: the i-th element, $V[i]$, may be empty or m_i (a proposal r-broadcast by p_i), depending on if the corresponding RABA instance RABA$_i$ decides 0 or 1, where $i \in [0..n-1]$. We just need to show that each replica p_k will output a set V' such that $V' = V$, i.e., $V[i] = V'[i]$ for $i \in [0..n-1]$.

If p_j outputs a set V, then all RABA instances either decide 0 or 1. According to Lemma F.1, we know all RABA instances must terminate. Due to the agreement property of RABA, these RABA instances decide the same values for p_k. Therefore, all RABA instances for p_k will terminate and decide the same values as p_j. Furthermore, the agreement of RBC instances guarantees that, $V'[i]$ will be r-delivered and $V[i] = V'[i] = m_i$ for all $i \in [0..n-1]$. ■

The above theorem immediately implies the usual agreement definition for BFT. We now prove a theorem implying efficiency and liveness.

Theorem F.3. *For each epoch, a set V containing at least $f + 1$ non-empty elements will be output.*

Proof. For simplicity, we assume $n = 3f + 1$. Conditioned on termination for all RABA instances (shown in Lemma F.1), we now bound the number of RABA instances that decide 1, which corresponds to the number of non-empty elements. We mainly use the biased termination property to prove the theorem for this proof.

According to the biased termination property, a RABA instance RABA$_i$ will decide 1, if $f + 1$ or more correct replicas propose 1. We now need to bound the number of RABA instances where less than $f + 1$ correct replicas propose 1.

A crucial observation is that a correct replica will propose 1 for at least $2f+1$ RABA instances, a fact guaranteed by RBC. All correct replicas will input 1 for $(2f+1)(2f+1)$ inputs for all RABA instances. There are at most $(3f+1)(2f+1)$ inputs for correct replicas. Hence, the total number of the 0 input from all correct replicas for all RABA instances is at most $(3f+1)(2f+1) - (2f+1)(2f+1) = 2f^2 + f$, while in the normal case, transactions from at least $\lceil \frac{n+f+1}{2} \rceil$ replicas will be delivered. Thus, the number of RABA instances that decide 0 is bounded by:

$$\frac{2f^2 + f}{f+1} < \frac{2f^2 + 2f}{f+1} = 2f.$$

The number of RABA instances that decide 1 is at least $f + 1$. ■

The above theorem implies that our new paradigm will a-deliver at least one proposal from a correct replica. The theorem also implies the liveness of our BFT protocol from the client perspective: a transaction from a correct client will be eventually a-delivered at some epoch.

G CONVERTING ABA TO RABA

Our strategy that converts ABA to RABA is generic. We now show how to convert a number of representative ABA protocols to RABA protocols, including the classic CKS ABA [17], CrainH ABA [25, 2nd algorithm], CrainL ABA [25, 1st algorithm], and Cobalt ABA [37]. As Cobalt, CrainH, and CrainL follow MMR, the proofs are similar to that for Pisa. Thus, we first focus on CKS [17] and provide a full proof for it. We then show in detail how to convert CrainH to RABA and briefly sketch how to do it for other protocols.

G.1 Converting CKS ABA to RABA

The pseudocode of CKS is shown in Figure 16. While the original CKS paper uses quite different notations, we use the same notations as other protocols presented in this paper. CKS uses a low-threshold $(n, f+1)$ threshold signature scheme (denoted $ts1$) and high-threshold $(n, n-f)$ threshold signature scheme (denoted $ts2$).

We use $ts1.share$ to represent a threshold signature share generated using $ts1$ and $ts1.sig$ to represent a threshold signature combined from $f + 1$ threshold signature shares. Similarly, we use $ts2.share$ to represent the threshold signature share for $ts2$ and $ts2.sig$ to present the threshold signature combined from $n - f$ shares. For each message, the threshold signature share is generated for a value v, the round number r, and the ABA instance ID sid.

The CKS protocol consists of three steps in round 0 (the first round) and two steps in each round starting from round 1. In round 0, at ln 07, every replica broadcasts a bval$_r(est_0, ts1.share)$ message, where est_0 is the proposed value and $ts1.share$ is a threshold signature share. At ln 08-09, upon receiving $2f+1$ valid bval$_r()$ messages, a replica sets a local parameter v as the majority value v received from bval$_r()$ messages and combines the threshold shares to sig. At ln 14, the replica then sends an aux$_r(v, sig, ts2.share)$ message

```
01 upon event $propose(sid, v_i)$
02   $r \leftarrow 0$
03   $est_0 \leftarrow v_i$
04   start round 0
05 round $r$
06   if $r = 0$
07     broadcast $\mathsf{bval}_r(est_0, ts1.share)$
08     upon receiving $2f+1$ $\mathsf{bval}_r()$ with $vals$
09       $v \leftarrow$ majority($vals$), $sig \leftarrow ts1.combine(shares)$
10   else
11     if exists $\mathsf{conf}_{r-1}(b, sig)$ s.t. $sig$ is a valid threshold signature
12       $v \leftarrow b$
13     else $v \leftarrow s_{r-1}$
14   broadcast $\mathsf{aux}_r(v, sig, ts2.share)$
15   upon receiving $n-f$ $\mathsf{aux}_r()$ messages where $vals$ is a set of values
     carried by these messages
16     if $vals = \{b\}$, $v \leftarrow b$, $sig \leftarrow ts2.combine(shares)$
17     else $v \leftarrow \perp$, $sig \leftarrow \mathsf{aux}_r()$ messages
18     broadcast $\mathsf{conf}_r(v, sig, ts2.share)$
19   upon receiving $n-f$ $\mathsf{conf}_r()$ messages where $vals$ is a set of
     values carried by these messages
20     if $vals = \{b\}$, $decide(sid, b)$
21     $s_r \leftarrow \mathsf{coin}_r$
22     $r \leftarrow r+1$
```

Figure 16: CKS ABA [17]. The code for replica p_i.

to all replicas, where *ts*2.*share* is a threshold signature share for the $\mathsf{aux}_r()$ message. Replicas then wait for the $\mathsf{aux}_r()$ messages (ln 15). If a replica receives $n-f$ $\mathsf{aux}_r()$ messages that contain only one valid value (ln 16), a replica combines the signature shares and obtain *sig*. The replica then broadcasts a $\mathsf{conf}_r(v, sig, ts2.share)$ messages where *ts*2.*share* is a threshold signature share for the $\mathsf{conf}_r()$ message (ln 18). If $\mathsf{aux}_r()$ message contains multiple values, the replica broadcasts a $\mathsf{conf}_r(\perp, \mathsf{aux}())$ message (combined from ln 17-18), where $\perp$ is a special symbol such that $\perp \notin \{0,1\}$, and $\mathsf{aux}_r()$ messages serve as justification for $\perp$. Finally, a replica waits for $n-f$ valid $\mathsf{conf}_r()$ messages (ln 19). If the replica receives $n-f$ $\mathsf{conf}_r(b,*,*)$, the replica decides b (ln 20). The replica then queries the common coin (ln 21) and enters the next round (ln 22).

Starting from round $r = 1$, replica do not have to broadcast $\mathsf{bval}_r()$ messages any more. Instead, at ln 11, a replica checks whether it has received a valid $\mathsf{conf}_{r-1}(b)$ message with a valid threshold signature *sig* (for *ts*2). If so, the replica sets v as b (ln 11-12). Otherwise, ln 13 sets v as s_{r-1}, the common coin value generated in the last round. Then the replica broadcasts an $\mathsf{aux}_r(v, sig, ts2.share)$ message (ln 14). Other steps remain the same as that in round 0.

After each replica decides, it continues to run the protocol for another round and terminate the protocol at the end of the $\mathsf{aux}_r()$ step after broadcasting the $\mathsf{conf}_r()$ message.

VABA. CKPS [16] proposed an approach that converts CKS to VABA, the pseudocode of which is shown in Figure 17. Note CKPS also defines biased validity, but in the context of external validity such that a predicate $Q(v, \pi)$ has to be verified by every replica before a vote for v can be accepted.

The VABA protocol CKPS proposes makes a few changes to make ABA achieve external validity and biased validity. First, every proposed value is associated with an external proof π, which needs

```
upon event $propose(sid, v_i, \pi)$
01   $r \leftarrow 0$
02   $\mathsf{coin}_0 \leftarrow 1$
03   $est_0 \leftarrow v_i$
04   start round 0
05 round $r$
06   if $r = 0$
07     broadcast $\mathsf{bval}_r(est_0, ts1.share, \pi)$
08     upon receiving $\mathsf{bval}_r()$ with invalid $\pi$
09       discard the message
10     upon receiving $2f+1$ $\mathsf{bval}_r()$ with $vals$
11       $v \leftarrow$ majority($vals$), $sig \leftarrow ts1.combine(shares)$
12   else
13     if exists $\mathsf{conf}_{r-1}(b, sig)$ s.t. $sig$ is a valid threshold signature
14       $v \leftarrow b$
15     else $v \leftarrow s_{r-1}$
16   broadcast $\mathsf{aux}_r(v, sig, ts2.share)$
17   upon receiving $n-f$ $\mathsf{aux}_r()$ messages where $vals$ is a set of values
     carried by these messages
18     if $vals = \{b\}$, $v \leftarrow b$, $sig \leftarrow ts2.combine(shares)$
19     else $v \leftarrow \perp$, $sig \leftarrow \mathsf{aux}_r()$ messages
20     broadcast $\mathsf{conf}_r(v, sig, ts2.share)$
21   upon receiving $n-f$ of $\mathsf{conf}_r()$ messages where $vals$ is a set of
     values carried by these messages
22     if $vals = \{b\}$, $decide(sid, b)$
23     if $r = 0$, $s_r \leftarrow 1$
24     else $s_r \leftarrow \mathsf{coin}_r$
25     $r \leftarrow r+1$
```

Figure 17: The VABA construction based on CKS ABA protocol [16].

to be verified before a replica accepts the proposed value. In the specific construction, a vote for 1 has to be associated with π, a threshold signature generated externally. For a vote for 0, π can simply be $\perp$. Namely, a $\mathsf{bval}_r()$ message with an invalid π will be discarded (ln 08-09). Second, the common coin in round 0 is set to 1 (ln 23).

RABA. We present a construction that converts CKS to a RABA protocol CKS-R, as shown in Figure 18. We make several changes. First, at ln 06-07, upon the *repropose*$(sid, 1)$ event, regardless of which round a replica is in, it broadcasts a $\mathsf{bval}_0(1, ts2.share)$ message where *ts*2.*share* is a threshold signature share for the $\mathsf{bval}_0()$ message. Second, the threshold signature scheme we use for the $\mathsf{bval}_r()$ messages is a $(n, n-f)$ scheme (ln 10-12). In other words, each replica generates a threshold signature share *ts*2.*share* in the $\mathsf{bval}_r()$ step (ln 10). In ln 11-12, each replica collects $n-f$ $\mathsf{bval}_r()$ messages with matching value to proceed to the next step. Third, the common coin of round 1 is set to 0 (ln 24-25).

Proof of Correctness for CKS-R. We show that our construction in Figure 18 satisfies the RABA security definitions.

Theorem G.1. ***(Validity)*** *If all correct replicas propose v and never repropose $\bar{v}$, then any correct replica that terminates decides v.*

Proof. Since a correct replica does not repropose a different value v', all replicas are able to receive $n-f$ $\mathsf{bval}_r(v)$ and obtain a valid threshold signature. Each replica only sends $\mathsf{aux}_r(1)$ with

```
01 upon event $propose(sid, v_i)$
02   $r \leftarrow 0$
03   $\mathsf{coin}_0 \leftarrow 1$
04   $est_0 \leftarrow v_i$
05   start round 0
06 upon event $repropose(sid, 1)$
07   broadcast $\mathsf{bval}_0(1, ts2.share)$
08 round $r$
09   if $r = 0$
10     broadcast $\mathsf{bval}_r(est_0, ts2.share)$
11     upon receiving $n - f$ $\mathsf{bval}_r(b)$
12       $v \leftarrow b$, $sig \leftarrow ts2.combine(shares)$
13   else
14     if exists $\mathsf{conf}_{r-1}(b, sig)$ s.t. $sig$ is a valid threshold signature
15       $v \leftarrow b$
16     else $v \leftarrow s_{r-1}$
17   broadcast $\mathsf{aux}_r(v, sig, ts2.share)$
18   upon receiving $n-f$ $\mathsf{aux}_r()$ messages where $vals$ is a set of values
carried by these messages
19     if $vals = \{b\}$, $v \leftarrow b$, $sig \leftarrow ts2.combine(shares)$
20     else $v \leftarrow \perp$, $sig \leftarrow \mathsf{aux}_r()$ messages
21     broadcast $\mathsf{conf}_r(v, sig, ts2.share)$
22   upon receiving $n - f$ $\mathsf{conf}_r()$ messages where $vals$ is a set of
values carried by these messages
23     if $vals = \{b\}$, $decide(sid, b)$
24     if $r = 0$, $s_r \leftarrow 1$
25     else $s_r \leftarrow \mathsf{coin}_r$
26     $r \leftarrow r + 1$
```

Figure 18: CKS-R: The RABA construction based on CKS.

a valid signature. Similarly, every replica only sends a $\mathsf{conf}_r(v)$ message and is able to decide v. ■

Theorem G.2. ***(Agreement)*** *If a correct replica decides v, then any correct replica that terminates decides v.*

Proof. We consider that a correct replica p_i decides v in round r. Replica p_i receives $n - f$ $\mathsf{conf}_r(v)$ messages. We show the case where another correct replica p_j decides v' 1) in round r; 2) in round $r' > r$.

First, if p_j decides v' in round r, it receives $n - f$ $\mathsf{conf}_r(v')$. Among $n - f$ replicas that send $\mathsf{conf}_r(v)$ and $n - f$ replicas that send $\mathsf{conf}_r(v')$, at least one sends both $\mathsf{conf}_r(v)$ and $\mathsf{conf}_r(v')$, a contradiction.

Second, if p_j decides in round r', it receives $n - f$ $\mathsf{conf}_{r'}(v')$. In other words, at least $n - f$ replicas receive $n - f$ $\mathsf{aux}_{r'}(v')$ messages. Among the messages, at least one includes a $\mathsf{conf}_{r'-1}(v')$ with a valid threshold signature. In other words, at least $n - f$ replicas sent $\mathsf{conf}_{r'-1}(v')$ in round $r' - 1$. Recursively, in round r, at least $n - f$ replicas sent $\mathsf{conf}_r(v')$ in round r. Therefore, a correct replica sends both $\mathsf{conf}_r(v)$ and $\mathsf{conf}_r(v')$, a contradiction. ■

Theorem G.3. ***(Biased validity)*** *If $f + 1$ correct replicas propose 1, then any correct replica that terminates decides 1.*

Proof. If $f+1$ correct replicas propose 1, none of correct replicas is able to collect $n - f$ $\mathsf{bval}_0(0)$. This is because correct replicas do not repropose 0 if they propose 1. If a replica receives $n - f$ $\mathsf{bval}_0(0)$ messages and assuming there are f faulty replicas, at least $n - 2f$ correct replicas have sent $\mathsf{bval}_0(0)$. Since $n \geq 3f + 1$, $n - 2f \geq f + 1$. This is a violation that at leat $f + 1$ replicas propose 1 since a correct replica will not repropose 0.

Also, each replica may repropose 1. In other words, all correct replicas will eventually receive $n - f$ $\mathsf{bval}_0(1)$ messages. In this case, all replicas will receive $n - f$ $\mathsf{bval}_0(1)$ and proceed to the next step.

Now assume that a correct replica p_j decides 0 in round 0. In this case, p_j receives $n - f$ $\mathsf{conf}_0(0)$ messages, which is impossible since it requires $n - f$ $\mathsf{bval}_0(0)$. We now only need to show the correctness by assuming that p_j decides in round $r > 0$. In this case, p_j receives $n - f$ $\mathsf{conf}_{r-1}(0)$. In other words, at least $n - f$ replicas broadcast $\mathsf{aux}_{r-1}(1)$ and obtain a valid threshold signature from round $r - 2$. Recursively, in round 0, at least $n - f$ replicas send $\mathsf{conf}_0(0)$. It is straightforward to see that in the first step of round 0, at least $n - f$ replicas sent $\mathsf{bval}_0(0)$. As shown previously, this is also impossible. ■

Theorem G.4. ***(Biased termination)*** *Let Q be the set of correct replicas. Let Q_1 be the set of correct replicas that propose 1 and never repropose 0. Let Q_2 be correct replicas that propose 0 and later repropose 1. If $Q_2 \neq \emptyset$ and $Q = Q_1 \cup Q_2$, then every correct replica eventually terminate.*

Proof. We distinguish three cases: 1) All replicas propose 1; 2) All replicas propose 0; 3) At least one correct replica proposes 1. We show correctness for the three cases.

1) Since correct replicas do not repropose 0, it is straightforward to see that all replicas will decide 1.

2) All correct replicas may repropose 1. In other words, every replica may receive $n - f$ $\mathsf{bval}_0(0)$ and $n - f$ $\mathsf{bval}_0(0)$. Each replica, however, only sends $\mathsf{aux}_r()$ message once with one value. In the next step, a replica makes a decision regardless of the values received from $\mathsf{aux}_r()$ messages. Similarly, a replica can proceed after it receives $n - f$ $\mathsf{conf}_r()$ messages regardless of the values. Therefore, the protocol can proceed to the next step. There are three sub-cases:

- A). None of the correct replicas collects $n - f$ $\mathsf{conf}_r()$ messages with a matching value; B) At least one but fewer than $f + 1$ correct replicas collect $n - f$ $\mathsf{conf}_r()$ messages with a matching value; C) At least $f + 1$ correct replicas collect $n - f$ $\mathsf{conf}_r()$ messages. In case A, all correct replicas use the common coin value as v and each replica sends $\mathsf{aux}_r(v)$. It is impossible that another corect replica sends $\mathsf{aux}_{r'}(v')$ with a valid threshold signature since it requires $n - f$ valid $\mathsf{conf}_{r-1}(v')$ messages. Therefore, all replicas will receive $n - f$ $\mathsf{aux}_r(v)$, send $\mathsf{conf}_r(v)$, and decide v.
- B). Some replicas may send $\mathsf{aux}_r(v)$ while other replicas use the common coin value. The probability that the common coin value is the same as the b value (if any) is 1/2. In other words, all replicas will decide with 1/2 probability. Otherwise, replicas may proceed to the next round. It is then straightforward to see that replicas will terminate the protocol.
- C). All correct replicas will receive at least one valid $\mathsf{conf}_r(v)$ in round $r+1$. This is because each replica collects $n - f$ $\mathsf{bval}_{r+1}(v)$ messages. Among the replicas that have sent $\mathsf{bval}_{r+1}(v)$ messages, at least $n - 2f$ are correct. We also know that at least $f + 1$

```
01 upon event $propose(sid, v_i)$
02   $r \leftarrow 0$
03   $\text{coin}_0 \leftarrow \bar{v}_i$
04   $support\_coin \leftarrow false$
05   $bin\_ptr_i[\bar{v}_i] \leftarrow S\_broadcast\ EST[1](\bar{v}_i, false)$
06   start the loop
07 round $r$
08   $r \leftarrow r+1$
09   $bin\_ptr_i[\bar{s}_{r-1}] \leftarrow S\_broadcast\ EST[r](\bar{s}_{r-1}, support\_coin)$
10   wait until $bin\_ptr_i[0] = true$ or $bin\_ptr_i[1] = true$
11     if $support\_coin$, $w \leftarrow s_{r-1}$
12     else if $bin\_ptr_i[0] = true$, $w \leftarrow 0$
13     else if $bin\_ptr_i[1] = true$, $w \leftarrow 1$
14     broadcast $\text{aux}_r(w)$
15    upon receiving $n-f$ $\text{aux}_r()$ with $vals$ such that for every value
      $v \in vals$, $bin\_ptr_i[v] = true$
16     $s_r \leftarrow \text{coin}_r$
17     if $vals = \{b\}$ and $b = s_r$
18       $v \leftarrow b$, $support\_coin \leftarrow true$
19       $decide(sid, b)$
20     else $vals = \{0,1\}$
21       $support\_coin \leftarrow true$
22     else
23       $support\_coin \leftarrow false$
24 operation $S\_Broadcast$ TAG$(v_i, should\_broadcast_i)$ {utility function}
25   $s\_value_i \leftarrow false$
26   if $should\_broadcast_i = true$, then broadcast TAG,S_VAL($v_i$)
27   return $s\_value_i$
28 upon receiving $f+1$ TAG,S_VAL($v$)
29     broadcast TAG,S_VAL($v$)
30 upon receiving $2f+1$ TAG,S_VAL($v$)
31     $s\_value_i \leftarrow true$
```

Figure 19: CrainH: Crain's ABA with high threshold common coins ([25, 2nd algorithm]). The code for replica p_i.

replicas sent $\text{conf}_r(v)$. There are in total $n-f+1$ correct replicas that sent $\text{bval}_{r+1}(v)$ and $\text{conf}_r(v)$. Therefore, if at least one corect replica fails to receive a valid $\text{conf}_r(v)$ message, at least one correct replica collects $n-f$ $\text{conf}_r(v)$ messages but does not send a $\text{bval}_{r+1}(v)$ with a valid signature, a contradiction.

3) If at least one replica proposes 1, all replicas may receive $n-f$ $\text{bval}_0(0)$ and/or $n-f$ $\text{bval}_0(0)$. It is also possible that a replica cannot collect $n-f$ $\text{bval}_0(0)$ or $n-f$ $\text{bval}_0(1)$ based on the proposed values. In this case, the external condition guarantees that all correct replicas will eventually broadcast $\text{bval}_0(1)$. In other words, replica will eventually proceed to the next step. Similar to case 2), all correct replicas will eventually terminate. ■

G.2 Converting Crain's ABA Protocols to RABA Protocols

We show how to convert both CrainL and CrainH [25] to RABA. As our measures to convert both protocols are similar, in this section, we focus on CrainH, the one that relies on high threshold common coins.

CrainH and CrainH-R. As illustrated in Figure 19, each round of CrainH involves an optional dispersal phase and an agreement phase. The dispersal phase (ln 09, which calls operations in ln 24-31) is similar to the $\text{bval}_r()$ phase in Pillar and Cobalt, where replicas also disperses a value if it has not previously done so but receives the value from $f+1$ replicas (ln 28-29). Upon receiving $n-f$ messages with the same value v, the replica updates its local parameter $bin\_ptr_i[v]$ to true (ln 09 and ln 30-31). In this phase, some correct replicas may disperse their values and some may not, depending on a boolean flag *support_coin*. The *support_coin* value is set to false by default in the first round and updated from the last round otherwise. In the agreement phase, replicas either broadcasts an $\text{aux}_r(1)$ or an $\text{aux}_r(0)$ message and only accepts an $\text{aux}_r(v)$ if $bin\_ptr_i[v]$ is true. After receiving $n-f$ $\text{aux}_r()$ messages, replicas query the common coin protocol coin_r and obtain s_r (ln 15-16). There are three cases: 1) If a replica receives $n-f$ $\text{aux}_r(b)$ and $b = s_r$, it decides b and sets *support_coin* to true (ln 17-19); 2) If a replicas receives both $\text{aux}_r(0)$ and $\text{aux}_r(1)$, it sets *support_coin* to true (ln 20-21); 3) Otherwise, the replica sets *support_coin* to false (ln 22-23). In all the cases, the replica continues to the next round.

```
01 upon event $propose(sid, v_i)$
02   $r \leftarrow 0$
03   $\text{coin}_0 \leftarrow \bar{v}_i$
04   $support\_coin \leftarrow false$
05   $bin\_ptr[\bar{v}_i] \leftarrow S\_broadcast\ EST[1](\bar{v}_i, false)$
06   start the loop
07 upon event $repropose(sid, v_i)$
08   broadcast $EST[1]$,S_VAL($v_i$)                {reproposal event}
09 round $r$
10   $r \leftarrow r+1$
11   $bin\_ptr_i[\bar{s}_{r-1}] \leftarrow S\_broadcast\ EST[r](\bar{s}_{r-1}, \neg support\_coin)$
12   if $r = 0$ and $v_i = 1$
13     $bin\_ptr_i[1] = true$
14   wait until $bin\_ptr_i[0] = true$ or $bin\_ptr_i[1] = true$
15     if $support\_coin$, $w \leftarrow s_{r-1}$
16     else if $bin\_ptr_i[0] = true$, $w \leftarrow 0$
17     else if $bin\_ptr_i[1] = true$, $w \leftarrow 1$
18     if $\text{aux}_r()$ has not been sent, broadcast $\text{aux}_r(w)$
19    upon receiving $n-f$ $\text{aux}_r()$ with $vals$ such that for every value
      $v \in vals$, $bin\_ptr_i[v] = true$
20     if $r = 1$, $s_r \leftarrow 1$
21     else $s_r \leftarrow \text{coin}_r$
22     if $vals = \{b\}$ and $b = s_r$
23       $v \leftarrow b$, $support\_coin \leftarrow true$
24       $decide(sid, b)$
25     else $vals = \{0,1\}$
26       $support\_coin \leftarrow true$
27     else
28       $support\_coin \leftarrow false$
29 operation $S\_Broadcast$ TAG$(v_i, should\_broadcast_i)$ {$S\_Broadcast$}
30   $s\_value_i \leftarrow false$
31   if $should\_broadcast_i = true$, then broadcast TAG,S_VAL($v_i$)
32   return $s\_value_i$
33 upon receiving $f+1$ TAG,S_VAL($v$)
34     broadcast TAG,S_VAL($v$)
35 upon receiving $2f+1$ TAG,S_VAL($v$)
36     $s\_value_i \leftarrow true$
```

Figure 20: CrainH-R: The RABA construction based on CrainH. The code for replica p_i.

In the first two cases, since *support_coin* is set to true, a replica will not disperse the values in the dispersal phase of the next round. In the third case, the replica disperses its value again in the next round.

Two ideas are crucial for the correctness of CrainH. First, a high threshold common coin is used instead of a regular common coin, ensuring that the values chosen by at least $f + 1$ correct replicas cannot be manipulated by an adversary. Second, the dispersal phase becomes optional such that a network scheduler cannot *make* correct replicas change their votes. The optional dispersal phase also makes the protocol enjoy a fast path starting from the second round.

We make the following changes to convert CrainH to a RABA protocol CrainH-R, as shown in Figure 20. Similar to our approach presented in this paper, we achieve this by revising the first round (round 1 in CrainH) only. In particular, upon reproposing 1, regardless of which round a replica is in, it disperses a $EST[1]$, S_VAL(1) message (ln 08). This message is called in the *S_Broadcast* function for the dispersal phase (ln 29-36). Second, upon proposing 1, a replica immediately sets $bin\_ptr_i[1]$ to true (ln 12-13). In other words, the replica will directly broadcast an $\text{aux}_r(1)$ message. Finally, the common coin value of the first round is set to 1 (ln 20). The first change ensures that, if $f + 1$ correct replicas propose 1, every correct replica will receive at least one $\text{aux}_r(1)$ such that no correct replica will decide 0. The second change ensures that all correct replicas can terminate the protocol if the predicate for biased termination is satisfied. Finally, the third change ensures that if at least $f + 1$ correct replicas propose 1, no correct replica will use 0 to enter round 1.

CrainL and CrainL-R. We now sketch CrainL and describe how to convert it to a RABA protocol called CrainL-R. CrainL has a *SBV_broadcast* function which involves a $\text{bval}_r()$ phase (dispersal phase) and an $\text{aux}_r()$ phase (agreement phase). In each round, the *SBV_broadcast* function is repeated twice. In the first time the function is called, every replica p_i inputs one value est_i and obtains a set $view_i[r_i, 1]$. The set may include one value v or both 0 and 1. If $view_i[r_i, 1]$ includes only one value v, p_i inputs v to the second *SBV_broadcast* instance. Otherwise, p_i inputs $\perp$. Similarly, at the end of the second *SBV_broadcast* instance, each replica obtains a set $view_i[r_i, 2]$. The set may include one value v, both v and $\perp$, or only $\perp$. In the first case, p_i decides v. In the second case, est_i (for the next round) is set as v. In the third case, est_i is set as the common coin. In CrainL, the common coin protocol is queried right after the second *SBV_broadcast* instance completes. In the revised version [28] that has the good-case-coin-free property, the common coin can be queried if p_i ensures that the first case does not apply.

We can convert CrainL to CrainL-R by making the following changes. First, upon reproposing 1, regardless of which round a replica is in, it calls the first *SBV_broadcast* instance and inputs 1. Second, upon proposing 1, each replica can directly sends all the messages for both *SBV_broadcast* instances, and sets $view_i[r_i, 1]$ and $view_i[r_i, 2]$ to 1. Finally, the common coin value it set to 1 for the first round.

G.3 Converting Cobalt ABA to RABA

We can also convert Cobalt ABA [37] to a RABA protocol. We also make three changes. First, upon proposing 1, each replica starts round 0 (the first round) and broadcasts a $\text{bval}_0(1)$. Each replica also immediately adds 1 to bin_values_0, and broadcasts an $\text{aux}_0(1)$ message. Second, upon reproposing 1, regardless of which round a replica is in, each replica broadcasts $\text{bval}_0(1)$. The replica adds 1 to bin_values_0 if it has not done so already. Furthermore, the replica broadcasts an $\text{aux}_0(1)$ if it has not broadcast any $\text{aux}_0()$ message. Finally, the common coin for round 0 is set to 1. The first change and the third one ensure that no correct replica will ever use 0 as est_1 if at least $f + 1$ correct replicas propose 1. The second change ensures that biased termination can be achieved.

H ADDITIONAL RELATED WORK

Much related work is discussed in the course of the paper. The section discusses additional related work.

Consensus and atomic broadcast. The BKR paradigm implies ABA and atomic broadcast are equivalent in asynchronous environments. Chandra and Toueg [20] demonstrate that multi-valued consensus is equivalent to asynchronous atomic broadcast. They also mention, informally, multi-valued Byzantine agreement and atomic broadcast are equivalent in asynchronous settings, a claim formally proven by Cachin, Kursawe, Petzold, and Shoup (CKPS) [16]. Correia, Neves, and Verissimo (CNV) show that multi-valued consensus (without using signatures) is also equivalent to atomic broadcast [22].

Parallel ABA. Some works (directly) study how to reduce the time complexity for n parallel ABA instances to a constant expected number of rounds (in some other contexts). Ben-Or's solution tolerates $f = O(\sqrt[4]{n})$ failures only [10]; Ben-Or and El-Yaniv provide a constant expected time protocol [11]. The protocol, unfortunately, would yield a prohibitively expensive BFT protocol with $O(n^4)$ message and communication complexity.

ABA and consensus as a building block. ABA can be used to build many core distributed computing abstractions, such as vector consensus [12], multi-valued Byzantine agreement [16], atomic broadcast (e.g., [12, 16, 29, 35, 38]), anonymous vector consensus [14], and many others. It crash-failure counterpart, consensus, is even more widely used in practice, such as terminating reliable broadcast, dynamic membership, and non-blocking atomic commit (see [15] for a summary of consensus-based systems and references therein).

Local-coin ABA. ABA protocols may rely on local coins. These protocols are information-theoretically secure but terminate in an expected exponential number of rounds [13]. RITAS [39], for instance, uses local-coin ABA to build asynchronous atomic broadcast using the protocol of Correia, Neves, and Veríssimo [23] that does not fall into the category of the BKR paradigm or the CKPS paradigm.

Asynchronous vs. partially synchronous BFT. Partially synchronous BFT systems never violate safety but achieve liveness when the network becomes synchronous [30]. In contrast, asynchronous BFT does not rely on any timing assumptions. It is shown that even for partially synchronous BFT protocols focusing on robustness [7, 21], their performance may drop 78%-99% [8] during failures or attacks. Moreover, partially synchronous protocols may achieve zero throughput with an adversarial network scheduler [38].

Asynchronous BFT protocols are intrinsically robust against performance and denial-of-service (DoS) attacks.

Asynchronous BFT with quantum safety. DAG-Rider [32] is a recent asynchronous BFT protocol achieving quantum safety but not quantum liveness. DAG-Rider has $O(1)$ running time and achieves $O(n^2 l + \lambda n^3 \log n)$ communication complexity. A more recent work of Das, Xiang, and Lin improves DAG-Rider with a communication of $O(n^2 l + \lambda n^3)$.

Tusk [26] implements a highly efficient asynchronous BFT protocol that significantly outperforms existing protocols, but it requires additional workers to help the reliable broadcast phase and thus is outside of the scope of the conventional BFT model we consider in this paper. The underlying technique, however, seem to work for all asynchronous BFT protocols known, as all such protocols require transmitting bulk data using reliable communication primitives.

Sub-optimal resilience. Assuming sub-optimal resilience, MiB [36] implements asynchronous BFT protocols with lower latency and higher throughput based on the BEAT library.

Mir-BFT. Mir-BFT [44] allows multiple leaders (replicas) to propose request batches independently, just as in asynchronous BFT protocols, but it works in partially synchronous environments.

Adaptive security. While many asynchronous BFT protocols achieve adaptive security, EPIC [35] is the first adaptively secure BFT protocol implemented. In the adaptive security model, the adversary can choose to corrupt replicas at any moment during the execution of the protocol. Prior protocols, such as SINTRA, HoneyBadgerBFT, BEAT, and Dumbo, achieve static security only, where the adversary needs to choose the set of corrupted replicas before the execution of the protocol. EPIC is built on top of BEAT yet with two significant differences: first, EPIC uses a hybrid transaction selection approach removing the need for threshold encryption used in BEAT; second, EPIC leverages a common-coin protocol with adaptive security [9]. However, the adaptively secure common coin protocol relies on expensive pairing-based cryptography. While achieving reasonable performance, EPIC is much less efficient in terms of both latency and throughput than BEAT; the fact, once again, substantiates the well-established view of favoring regular cryptography (e.g., elliptic curve) over pairing-based cryptography.

While one can make protocols in the BKR paradigm adaptively secure as shown in EPIC [35], it is challenging to build practical BFT protocols with adaptive security from CKPS or Dumbo. In particular, the CKPS paradigm uses expensive threshold cryptography extensively, and it would be inefficient to replace these cryptographic operations using much more expensive adaptively secure cryptography [9].

Communication-efficient RBC protocols. Some recent constructions have improved the RBC protocols asymptotically or concretely [5, 27]. These protocols may benefit practical BFT protocols implemented in the bandwidth usage.